



DET NORSKE  
**VERITAS**

DEVELOPMENT OF AN ANALYTICAL MODEL  
OF THE HYDRODYNAMIC ADDED MASS OF  
A SHIP RAMMING ICE

Volume II: Main Report  
TP 8392E

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| 17. Résumé<br><p>L'objectif de l'étude susmentionnée consistait à élaborer un modèle analytique étalonné dans le but d'évaluer la masse ajoutée d'un navire heurtant les glaces, permettant ainsi de mieux comprendre l'action réciproque entre le navire et les glaces. Les travaux ont comporté quatre approches distinctes, soit: modélisation analytique, analyse des données vraie grandeur, essais sur maquettes et analyse selon la méthode des éléments finis. Seule la collision à angle droit entraînant les trois mouvements fondamentaux a été présumée. La modélisation analytique a comporté une étude de la documentation et l'élaboration d'un premier modèle pour le calcul des coefficients des trois mouvements fondamentaux. En outre, les effets de la proximité de la glace et du fond marin ont été étudiés. Une méthode a été mise au point afin d'analyser les données d'essais de charge des glaces réalisés en 1983 à l'aide du CANMAR KIGORIAK. Un programme d'essai exhaustif sur maquettes 1:50 du M.V. ARCTIC et 1:22,5 du KIGORIAK a permis d'obtenir une source considérable de données touchant l'oscillation libre des maquettes, les charges de choc et les scénarios de charge des glaces. En dernier lieu, un modèle d'ensemble tridimensionnel à éléments finis a été mis au point; il permet de déterminer la masse ajoutée de n'importe quel navire, avec définition arbitraire des limites représentant la proximité du fond marin et de la glace, l'épaisseur et la géométrie des glaces et la forme de la coque.</p> |                                                                         |                                                                                                             |                                           |
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## ABSTRACT

Ice loads are believed to be the governing factor in designing icebreaking ships. The ice load is significantly influenced by the magnitude of the hydrodynamic added mass. The objective of the work carried out herein was to develop a calibrated analytical model for the evaluation of the added mass of a ship ramming ice in order to provide a better understanding of the ship ice interaction process.

The work entailed four different approaches aimed at determining the added mass. These were: analytical modelling, full scale data analysis, model testing, and finite element modelling. Only symmetric ramming was assumed effecting surge, heave, and pitch rigid body motions. The first three vertical flexible modes were to be considered wherever possible (i.e modes having 2-, 3- and 4-nodes).

The analytical modelling task involved the review of the literature and the development of a preliminary model for calculating the surge, heave and a pitch added mass coefficient and the effects of bottom proximity and ice proximity were studied. A procedure was developed for analysing the 1983 full scale ramming test data of the Canmar Kigoriak. The procedure was integrated into previous data analysis software. The results obtained were reasonable except for errors due to numerical instabilities which occurred while dividing the impact forces by very small accelerations. The inaccuracies of the method of measuring forces lead to nonzero forces at the time and location of the small accelerations. However with careful analysis, it was possible to deduce some useful results from the full scale data with respect to the vertical added mass.

A comprehensive model testing program of an M.V. Arctic 1:50 model and a Kigoriak 1:22.5 model was carried out. Extensive data was collected covering free oscillations, impulsive loading, and ramming scenario. These data have only been partially analysed herein (approximately 30 per cent). This analysis resulted in accurate evaluation of added mass coefficients during the free oscillation phase. Further work is recommended in order to realise the full benefit from these data.

Finally, a comprehensive three-dimensional finite element model was developed using Det norske Veritas computer program SESAM. The model employed a beam representation of the hull girder stiffness. The geometry of the hull surface was carefully modelled using surface boundary elements rigidly connected to the beam nodes. The fluid was modelled using brick elements and assuming ideal incompressible fluid flow. This model is capable of predicting the added mass of any ship with arbitrary definition of the boundaries representing the bottom and ice proximities, the ice thickness, ice geometry and hull shape. The developed model requires further computer runs in order to achieve a more accurate determination of the parametric relationships on which the added mass is dependent.

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## NOMENCLATURE

|                      |                                                                           |
|----------------------|---------------------------------------------------------------------------|
| $a$                  | dimension between load location and support                               |
| $a$                  | subscript used to indicate added mass                                     |
| $A$                  | cross sectional area or body projected area normal to direction of motion |
| $A_x$                | submerged cross sectional area                                            |
| $A_{wl}$             | waterline area                                                            |
| $b$                  | dimensions between load and support                                       |
| $b$                  | half width of ship section                                                |
| $B$                  | beam of ship                                                              |
| $B_p$                | bottom proximity                                                          |
| $c$                  | width of load                                                             |
| $c$                  | damping coefficient                                                       |
| $C_x, C_z, C_\theta$ | damping coefficients for surge, heave and pitch motions                   |
| $C_M$                | section: area coefficient                                                 |
| $C_{wl}$             | waterline area coefficient                                                |
| $C_B, CB$            | block coefficient                                                         |
| $D$                  | depth                                                                     |
| $e$                  | exponential constant (2.7183)                                             |
| $e$                  | bottom proximity                                                          |
| $E$                  | modulus of elasticity                                                     |
| $f_i$                | natural frequency of mode $i$ ( $H_z$ )                                   |
| $F$                  | drag force                                                                |
| $F_x, F_z$           | horizontal and vertical components of the bow impact force                |
| $F_{max}$            | maximum impact force                                                      |
| $g$                  | gravitational acceleration                                                |
| $H_{ice}$            | ice thickness                                                             |
| $I$                  | moment of inertia of the hull girder                                      |
| $I_o$                | pitch mass moment of inertia                                              |
| $k$                  | imperical constant                                                        |
| $k$                  | spring stiffness                                                          |
| $K_{bp}$             | correction factor for bottom proximity effect                             |
| $K_{fm}$             | correction factor for frequency of motion effect                          |

|                 |                                                                  |
|-----------------|------------------------------------------------------------------|
| $K_{fs}$        | correction factor for free surface effect                        |
| $k_{sp}$        | correction factor for speed effect                               |
| $K_{wp}$        | correction factor for wall proximity effect                      |
| $k_{\theta}$    | rotational stiffness for pitch motion                            |
| $K$             | stiffness matrix                                                 |
| $L$             | ship length                                                      |
| $L$             | beam length                                                      |
| $L_{pb}$ or LBP | length between perpendiculars                                    |
| $L_b, L_s$      | distance from C.G. to F.P. and A.P., respectively                |
| $L_{OA}, LOA$   | ship overall length                                              |
| $M_o$           | ship or model mass                                               |
| $m_a$           | added mass                                                       |
| $m_{az}$        | unit mass in z-direction                                         |
| $m_s$           | unit mass of ship                                                |
| $Ma$            | added mass matrix                                                |
| $M$             | mass matrix                                                      |
| $M_L$           | left support bending moment                                      |
| $M_R$           | right support bending moment                                     |
| $M_{ii}$        | average bending moment in segment i                              |
| $n$             | distance along normal to surface                                 |
| $n, N$          | subscript to indicate the direction of the normal to the surface |
| $p$             | pressure in fluid                                                |
| $p$             | subscript for pitching motion                                    |
| $q$             | distributed load per unit ship length                            |
| $q_h$           | change in hydrostatic unit load                                  |
| $q_i$           | inertia load per unit ship length                                |
| $T$             | ship draft                                                       |
| $T_D$           | period of vibration                                              |
| $T_s$           | section draft                                                    |
| $u, v, w$       | fluid velocities in x, y and z directions                        |
| $U$             | body speed                                                       |
| $V$             | shear force                                                      |
| $V$             | volume of fluid domain                                           |

|                        |                                                        |
|------------------------|--------------------------------------------------------|
| $x$                    | coordinate axis in surge direction                     |
| $x$                    | distance to floe from ship's F.P.                      |
| $x$                    | coordinate axis for a beam element                     |
| $X, \dot{X}, \ddot{X}$ | displacement, velocity and acceleration in x direction |
| $X_i$                  | eigen mode shape i                                     |
| $x, y, z$              | cartizian coordinate axes                              |
| $z$                    | coordinate axis in heave direction                     |
| $z$                    | vertical displacement or motion                        |
| $\dot{z}, \ddot{z}$    | velocity and acceleration in z direction               |
| $\ddot{z}_b$           | bow vertical acceleration                              |
| $\ddot{z}_G$           | C.G. vertical acceleration                             |
| $\ddot{z}_s$           | stern vertical acceleration                            |
|                        |                                                        |
| $\lambda$              | stem angle                                             |
| $\lambda_w$            | water unit weight                                      |
| $\delta$               | logarithmic decrement                                  |
| $\Delta$               | ship displacement                                      |
| $\rho$                 | water density                                          |
| $\theta$               | pitch angle                                            |
| $\theta$               | subscript for pitch direction                          |
| $\lambda$              | added mass coefficient                                 |
| $\lambda$              | added mass coefficient before correction               |
| $\mu$                  | coefficient of friction                                |
| $\phi$                 | flow potential                                         |
| $\nabla$               | volume of body                                         |
| $\nabla^2$             | Laplacian Operator                                     |
| $\omega$               | undamped circular frequency (rad/s)                    |



## 1.0 INTRODUCTION

### 1.1 General

The evaluation of the forces exerted on a ship by various ice features, as it navigates through ice covered waters is an essential part of the design process since these forces substantially exceed those encountered in an open water environment. The past decade has witnessed a concentrated effort in this direction. The following areas of development have been addressed by several researchers:

- (a) Full scale testing,
- (b) Model testing, and
- (c) Analytical modelling.

Dedicated full scale ship tests in ice date back to the Manhattan of 1966. The following vessels have also been involved in dedicated tests for the measurement of ice forces

- (a) The Canmar Kigoriak (7, 12, 13, 14)\*
- (b) The M.V. Arctic (46)
- (c) The Robert Lemeur (7, 14)
- (d) The Polar Sea (9, 10)
- (e) The Polar Stern
- (f) The Kalvik
- (g) The Urho (49).

Several methods for measuring local and global ice forces have been successfully applied (14, 46) yielding an extensive data base in this area which will prove extremely useful in the development of the new generation of icebreaking vessels.

Model testing has also been advanced considerably in the past decade although there still exists several unresolved issues with respect to scaling laws due to the inevitable distortions that exist in modelling the ice and ship properties. Significant efforts are needed in advancing these model testing techniques.

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\* Numbers in brackets refer to the references listed at the end of the text.

Finally, in order to be able to scale the results obtained from full scale tests of existing vessels and model test results to predict ice loads on larger future vessels, the physical phenomenon of ship ice interaction has to be understood. This understanding would then be manifested in the form of the development of a ship ice interaction model. The elements of such a model are shown in Fig. 1 and include:

1. Definition of ship properties such as hull girder stiffness, local structure geometry and stiffness and ship mass distribution.
2. An estimate of the properties of ice relevant to the local and global interaction such as ice strength, which is a function of the magnitude of the confining pressure, ice temperature, size, and salinity.
3. The added mass distribution in the rigid body and flexible hull vibration modes.
4. A suitable idealization of the ship hull, the ice and the water during the interaction.
5. An algorithm for determining the dynamic equilibrium position in the time domain or an equivalent frequency domain analysis.

The authors are aware of the following ship ice interaction analytical model developments that took place in recent years:

1. Det norske Veritas model (18). This model takes into account the flexibility of the hull girder and is a time domain simulation. The ship is idealized as a beam element.
2. Arctec Canada model (10).
3. Hitach-Zosen model (31).
4. Canmar model (6).
5. VTT model (46).

A common weakness in all such models lies in the definition of the magnitude and distribution of the added mass. The resulting ice forces are significantly sensitive to

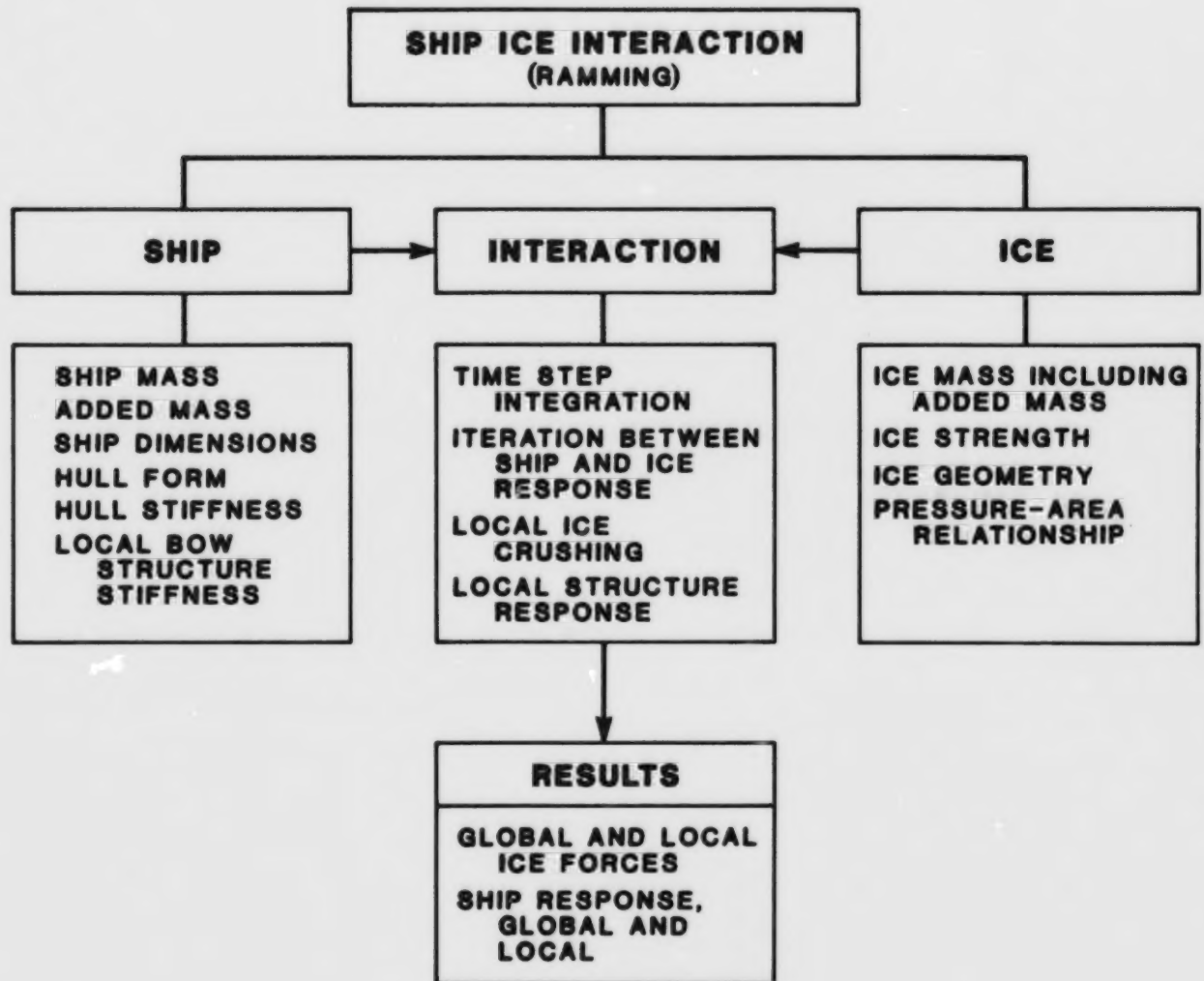


Fig. 1 Elements of Ship Ice Interaction Model

this parameter. Hence the need for an accurate method for evaluating the added mass effect in ship ice interaction.

In addition to the development of ship ice interaction models which utilize computers in order to carry out the analysis, several analytical and semi-empirical formulas have been proposed (15, 20, 21). Such formulas are more suited for regulatory purposes although they suffer from significant limitations due to their inability to account for effects of all important parameters. Such formulas are also sensitive to the magnitude of the added mass.

The preliminary analysis of the Canmar Kigoriak (6, 13) full scale data sponsored by the Canadian Coast Guard, has shown that the added mass is a time dependent parameter that varies throughout the ship ice interaction process. It was therefore recognized that an arbitrary assumption of the added mass is no longer adequate and it is through this realization that Det norske Veritas submitted its unsolicited proposal to the Department of Supply and Services Canada in 1985 to execute a comprehensive study directed towards the evaluation of added mass during ice ramming. The contract was awarded in July 1986 and the results of the work are presented in this report. The work was carried out by Det norske Veritas (Canada) Ltd. in cooperation with the Transportation Development Centre. The following organizations participated in the project:

1. The National Research Council of Canada, Institute for Marine Dynamics.
2. Offshore Research Ltd., Vancouver.
3. Canadian Marine Drilling Ltd., Calgary.
4. Arctic Research and Development, Bragg Creek.

## 1.2 Objective

As stated in the Contract work statement, the overall objective of this work is to contribute to the promotion of the development of advanced ice transitting performance modelling techniques, including full scale data acquisition, and correlation with related system concepts.

The specific project objective is to develop a calibrated analytical model for the evaluation of the hydrodynamic added mass of a ship ramming ice, in order to provide a better understanding of the ship ice interaction process.

### 1.3 Scope

This project deals only with the ice ramming scenario of icebreaking. In addition, it is assumed that the ramming is symmetrical and hence only the vertical modes of vibration are considered. Added mass coefficients are sought for the surge, heave and pitch rigid body modes as well as the first three flexible vertical modes, i.e the 2, 3 and 4-node modes.

Six tasks were identified against which the progress was measured. These tasks are briefly described below:

#### Task 1:    **Preparations and Analytical Modelling**

This task entailed the following:

- (a) Literature Review.
- (b) Summary of Theoretical Methods and Available Data.
- (c) Significant Parameters Identification
  - Hull form
  - Water depth
  - Ice edge proximity
  - Impact velocity
  - Ice strength
  - Hull girder stiffness.
- (d) Analytical Model Development
  - Three rigid body motions (surge, heave and pitch)
  - First three vertical flexural modes
  - Finite element and conformal mapping techniques
  - Capability to examine effect of significant parameters.

- (e) Application of Model to Experiment
  - Model limitations
  - Computer programs
  - Comparisons to be sought.

**Task 2:     Analysis of Full Scale Data of Kigoriak**

This task entailed the following:

- (a) Kigoriak Parameters and Full Scale Tests Overview
  - Draft, C.G., mass distribution, moment of inertia, displacement, section area and modulus and location of neutral axis.
  - Describe full scale test conditions.
- (b) Selection of Representative Rams.
- (c) HPL Program for Derivation of Added Mass from Full Scale Data.
- (d) Analyse selected Rams using Analytical Model.

**Task 3:     Model Tests Planning**

This task included the following:

- (a) Describe M.V. Arctic Parameters
  - Draft, C.G., mass distribution, moment of inertia and displacement.
- (b) Experiment Procedure
  - Initial disturbance, forced motions, and ramming tests
  - Dynamic response of force blocks
  - Ice representation: geometry, strength, support.
- (c) Data Recording and Output Procedures.
- (d) Extraction of Added Mass from Measurements.
- (e) Detailed Test Programs
  - Kigoriak and Arctic
  - Parameter variations.
- (f) Model Acquisition and Subcontracting Test Facility.



**Task 4:     Conduct Model Tests**

This task included the following:

- (a)    Install and Check Apparatus and Models
  - Calibrate recording devices
  - Preliminary experiments.
- (b)    Test Program Execution.
- (c)    Analyse Test Data.

**Task 5:     Data Comparisons and Model Refinements**

This task entailed the following:

- (a)    Kigoriak Added Mass and Damping Coefficients from analytical and physical models.
- (b)    M.V. Arctic Added Mass and Damping Coefficients from analytical and physical models.
- (c)    Compare Kigoriak Results with Full Scale Data.
- (d)    Compare Kigoriak and M.V. Arctic Results.
- (e)    Summarize the Results and refine Analytical Model.

**Task 6:     Reporting**

This task included the preparation of

- (a)    Interim Report No. 1
  - Results of Tasks 1 and 3 (a) - (e)
  - Progress 2 (a) - (d).
- (b)    Interim Report No. 2
  - Results of Tasks 4 (a) and 4 (b)
  - Progress 4 (c), 2 (a) - (d).
- (c)    Draft Final Report.
- (d)    Final Report.



This report is the final report on this project. The report provides a full description of the work carried out and should not require any reference to progress or interim reports. This report can not however include all collected and analysed data due to space limitations. The data constitutes thousands of pages which are stored in the form of magnetic tapes and diskettes for future reference. Salient data and results are however presented herein and should give a reasonable representation of the full data base.

Following this introduction, three sections are presented. Section 2 entitled Methodology gives the theoretical background and the developed methodologies for calculating added mass using analytical and finite element models, full scale data, and model tests. Section 3 presents the salient results obtained from the various formulations as well as a comparison between the calculated and measured added mass coefficients. Section 4 gives a detailed presentation of the conclusions drawn from the results of this project and makes recommendations for further work which would lead to the development of a general analytical model for the prediction of added mass for any vessel operating in Arctic waters. This model should be presented in a format suitable for direct application by ship designers and operators. This report consists of three volumes, Vol. I is a summary report, Vol. II is this main report and Vol. III\* includes the appendices. Four Appendices are given in Volume III which include further results from model tests, full scale data analysis, finite element analysis, and analytical modelling. These appendices are:

- A Analytical Modelling - Subcontractor's Report
- B Model Tests - Subcontractors Reports and Test Results
- C Analysis of M.V. Kigoriak Full Scale Data
- D Finite Element Modelling

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\* Unpublished. Available for consultation at Transportation Development Centre.

## 2.0 METHODOLOGY

The concept of added mass dates back to Lamb (25) "A treatise on the mathematical theory of the motion of fluids", published in 1932. Much work has been done since then mainly on vibrating ship-like bodies. Little work has been carried out on added mass for icebreaking ships impacting ice. The work presented herein aims at the bridging of this gap and the development of reliable methods for the prediction of added mass for icebreaking ships.

Following a brief introduction to the concept of added mass, this section describes the various analysis techniques employed and developed for the prediction of added mass coefficients in ship ice ramming using:

- (a) Analytical models,
- (b) Full scale data,
- (c) Model tests, and
- (d) Finite element modelling.

### 2.1 Introduction

Whenever acceleration is imposed on a fluid flow by the acceleration of a body, additional fluid forces will act on the surfaces in contact with the fluid. These fluid inertial forces have to be proven to be of great significance in many ocean engineering problems. In icebreaking, the forces that have to be resisted by the hull structure of the ship are significantly influenced by the magnitude of these inertia forces in the fluid.

The simplest view of the phenomenon of "added mass" is, according to Brennen (4), that it determines the necessary work done to change the kinetic energy associated with the motion of the fluid. The kinetic energy of fluid motion associated with the motion of a body through it can be expressed as:

$$T = \frac{\rho}{2} \int_V (u^2 + v^2 + w^2) dV \quad (1)$$

where  $T$  = kinetic energy of the fluid domain of volume  $V$ ,  $u$ ,  $v$  and  $w$  are the cartesian velocities of the flow of an element of volume  $dV$  of the fluid, and  $\rho$  is the fluid density assuming the fluid to be incompressible. If the body moves with a constant speed  $U$  through the fluid, then the kinetic energy  $T$  remains constant with time. Equation 1 can be written in terms of the velocity of the body  $U$  as follows:

$$T = \frac{\rho}{2} I U^2 \quad (2)$$

where

$$I = \int_V \left( \frac{u^2 + v^2 + w^2}{U^2} \right) dV \quad (3)$$

is an invariant number. This is true for some fluid flow solutions such as potential flow and low Reynolds number Stokes flow.

If the body accelerates or decelerates, the kinetic energy in the fluid will change as  $U$  changes requiring the work done by the body on the fluid to increase or decrease accordingly. The additional work is experienced by the body in the form of additional drag,  $F$ , such that the rate of additional work done is equal to the rate of change in the kinetic energy, i.e:

$$- F \cdot U = \frac{dT}{dt} \quad (4)$$

The force is therefore given as:

$$F = - \frac{1}{U} \frac{dT}{dt} = - \rho I \dot{U} \quad (5)$$

This force has the same form and sign as that required to accelerate the body itself, i.e.

$$F = m_a \dot{U} \quad (6)$$

where

$$m_a = - \rho I = - \rho \int \left( \frac{u^2 + v^2 + w^2}{U^2} \right) \quad (7)$$

is an equivalent mass added to that of the body, hence the terminology "added mass". Therefore, to give a ship an acceleration  $\dot{U}$ , the following force has to be applied to it by the propulsion system,

$$F_{TOT} = (M_s + m_a) \dot{U} \quad (8)$$

where  $M_s$  is the mass of the ship itself. Equation 8 can be written as

$$F_{TOT} = M_s (1 + \lambda) \dot{U} \quad (9)$$

where  $\lambda$  is the added mass coefficient

$$\lambda = \frac{M_a}{M_s} \quad (10)$$

When a ship rams an ice flow, several phases of the impact process are identified

- (a) Approach,
- (b) Impact, and
- (c) Sliding or beaching.

Figure 2 schematically depicts these phases of impact. Figure 3 shows the vibration modes of interest in this investigation assuming perfectly symmetrical impact, which gives rise to vertical plane modes only; namely, the surge, heave and pitch and the

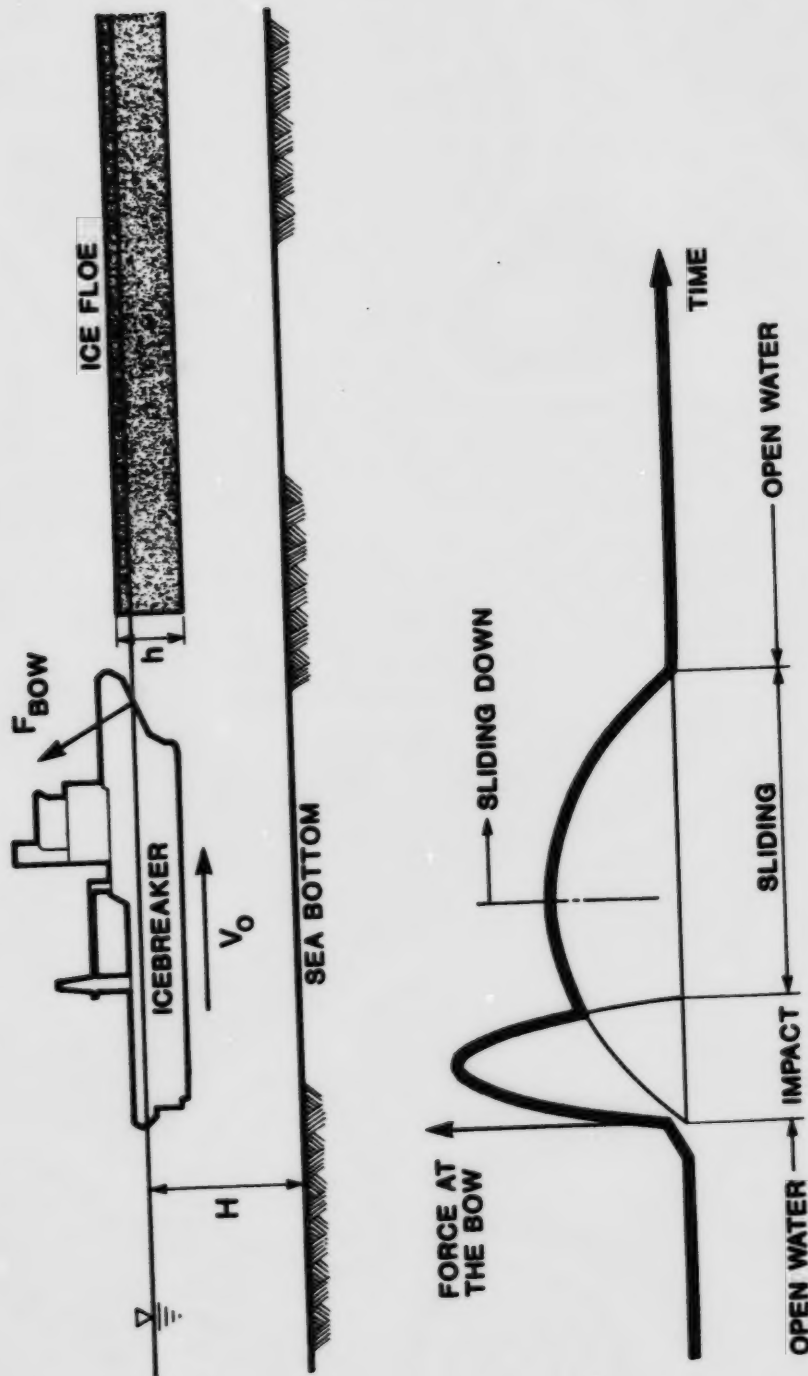


Fig. 2 Phases of Impact

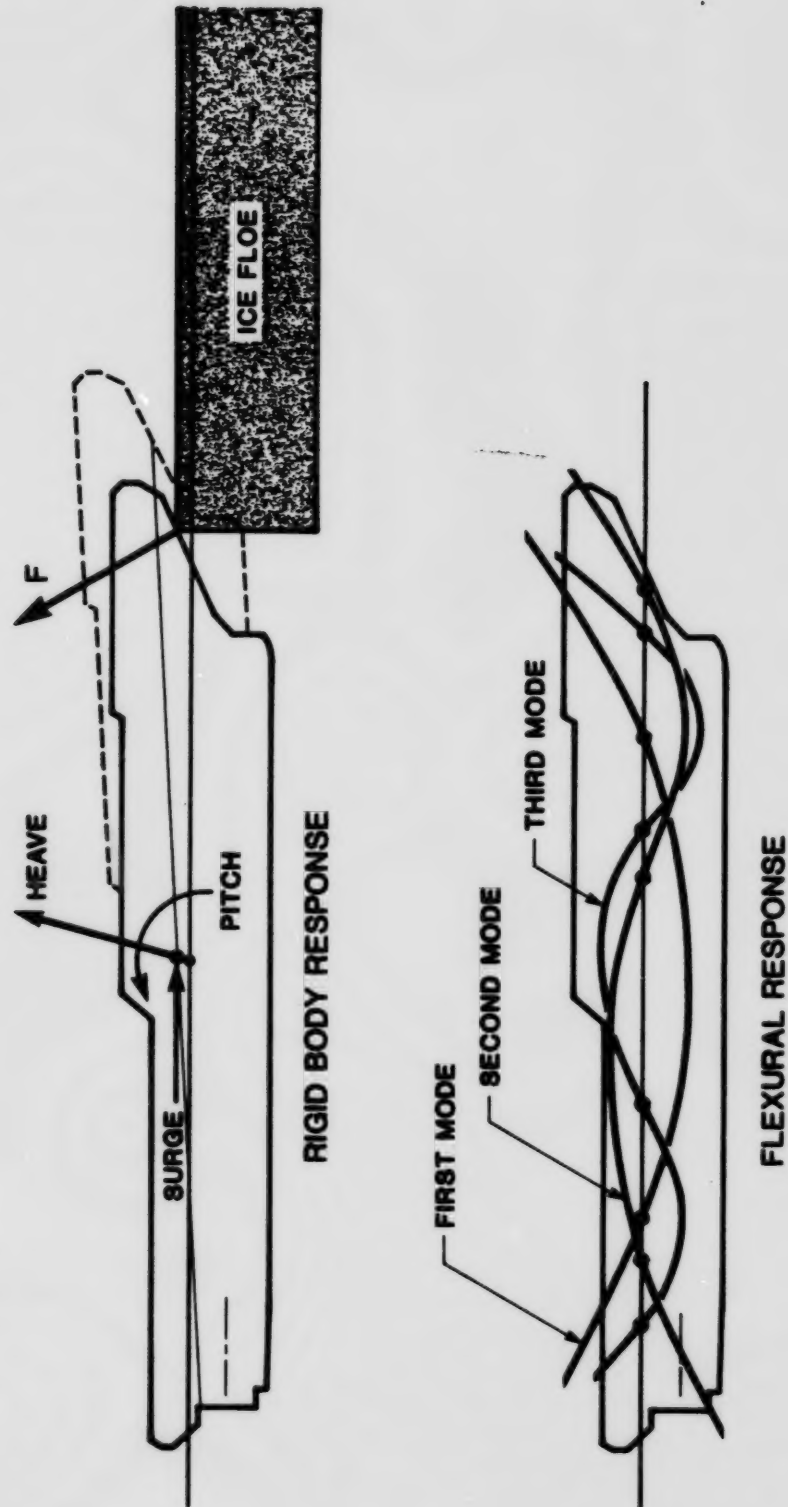


Fig. 3 Rigid Body and Vibration Modes of Interest

vertical flexible hull modes. Only the first three flexible modes are included in the present analysis, since it is a safe assumption that most of the strain energy/kinetic energy exchange will occur in these relatively low frequency modes.

The effect on the added mass of the following parameters are considered; Fig. 4,

- (a) Bottom proximity,
- (b) Floe proximity,
- (c) Ice thickness,
- (d) Hull geometry,
- (e) Ice strength, and
- (f) Hull girder stiffness.

The following four subsections describe the methods employed to calculate the added mass coefficients in the various impact phases using analytical formulations, full scale data, model tests, and finite element modelling of the ship, fluid and ice boundaries.



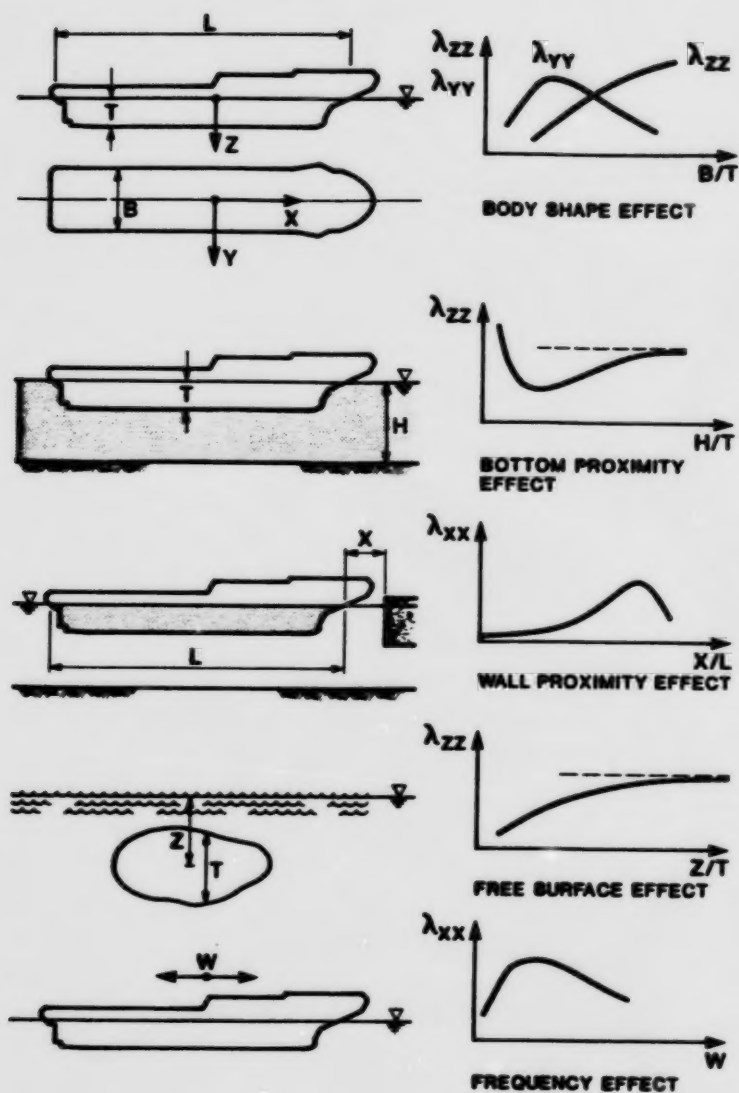


Fig. 4 Parameters Influencing Added Mass

## 2.2 Analytical Modelling

The development of an analytical model for the evaluation of added mass coefficients for a ship impacting ice is the major objective of the work carried out herein. It should however be recognized that this development entails both analytical formulations as well as numerical finite element modelling techniques. It was therefore decided to separate the two aspects with this subsection devoted to the development of theoretical expressions for added mass and Section 2.5 dealing with the application of the finite element modelling technique. Supplementary information is provided in Appendix A regarding the analytical modelling work. the appendix includes the final report by Arctic Research and Development Ltd. which was subcontracted to carry out the task of obtaining theoretical expressions for calculating added mass coefficients.

### 2.2.1 Rigid Body Motions

For symmetrical ramming the only three rigid motions possible are surge, heave and pitch. The general equations of motion representing these motions are:

$$m_0 (1 + \lambda_x) \ddot{x} + C_x \dot{x} + K_x x + m_0 \lambda_{xz} \ddot{z} + C_{xz} \dot{z} + K_{xz} z + I_0 \lambda_{x\theta} \ddot{\theta} + C_{x\theta} \dot{\theta} + K_{x\theta} \theta = F_x(t) \quad (11a)$$

$$m_0 (1 + \lambda_z) \ddot{z} + C_z \dot{z} + K_z z + m_0 \lambda_{zx} \ddot{x} + C_{zx} \dot{x} + k_{zx} x + I_0 \lambda_{z\theta} \ddot{\theta} + C_{z\theta} \dot{\theta} + K_{z\theta} \theta = F_z(t) \quad (11b)$$

$$I_0 (1 + \lambda_\theta) \ddot{\theta} + C_\theta \dot{\theta} + K_\theta \theta + m_0 \lambda_{\theta x} \ddot{x} + C_{\theta x} \dot{x} + k_{\theta x} x + m_0 \lambda_{\theta z} \ddot{z} + C_{\theta z} \dot{z} + K_{\theta z} z = M(t) \quad (11c)$$

where  $m_0$  is the mass of the ship,  $I_0$  is the mass moment of inertia about the Y-axis going through the centre of gravity of the ship,  $\lambda$  are added mass coefficients,  $C$  are damping coefficients and  $K$  are "stiffness" or restoration coefficients. The subscripts  $x$ ,  $z$  and  $\theta$  refer to the surge, heave and pitch motions, respectively. The added mass coefficients in Eq. 11 are

$$\begin{aligned}\lambda_x &= \text{surge added mass coefficient} \\ \lambda_z &= \text{heave added mass coefficient} \\ \lambda_\theta &= \text{pitch added mass coefficient} \\ \lambda_{xz} &= \text{coupling added mass coefficient of surge on heave or vice versa} \\ &\quad (\lambda_{xz} = \lambda_{zx}) \\ \lambda_{x\theta} &= \text{coupling added mass coefficient of surge on pitch and vice versa} \\ &\quad (\lambda_{x\theta} = \lambda_{\theta x}) \\ \lambda_{z\theta} &= \text{coupling added mass coefficient of heave on pitch and vice versa} \\ &\quad (\lambda_{z\theta} = \lambda_{\theta z})\end{aligned}$$

The values proposed in the literature (32, 36) for the above coefficients in open water are summarized in the following:

$$\begin{aligned}\lambda_x &= 0.075 \\ \lambda_{xz} &= \lambda_{x\theta} = 0.0 \\ \lambda_{z\theta} &= -0.013\end{aligned}\tag{12}$$

$$\lambda_z = 0.78 \frac{B}{T} \frac{C_{wl}}{C_B} \frac{1}{1 + C_{wl}} \cdot \frac{1}{1 + 0.8 B/L}\tag{13}$$

$$\lambda_\theta = 0.926 \frac{B}{T} \frac{C_{wl}}{C_B} \frac{1}{(3 - 2 C_{wl})(3 - C_{wl})} \frac{1}{1 + 0.8 B/L}\tag{14}$$

where  $B$  is ship beam,  $T$  is its draft,  $C_{wl}$  and  $C_B$  are the waterline and block coefficients, respectively.

The development given in Appendix A of general formulations for added mass show that the added mass coefficient  $\lambda$  can be expressed as

$$\lambda = C \cdot A^{3/2} / \nabla$$

where  $C$  is a constant,  $A$  is the projected area normal to the direction of motion, and  $\nabla$  is the volume of the body. These formulations lead to the following expressions for the basic added mass coefficients in surge, heave and pitch for open water conditions:

$$\hat{\lambda}_x = 0.788 \left( \frac{B/T}{1 + (B/T)^2} \right)^{1/2} \left( \frac{C_M B T}{C_B LBT} \right)^{3/2} \quad (15)$$

$$\hat{\lambda}_z = 0.394 \left( \frac{L/B}{1 + (L/B)^2} \right)^{1/2} \left( \frac{C_{wl} LB}{C_B LBT} \right)^{3/2} \quad (16)$$

$$\hat{\lambda}_\theta = 0.0825 \frac{1 + C_{wl}}{(3 - 2 C_{wl})(3 - C_{wl})} \cdot \left( \frac{\hat{\lambda}_z \cdot \Delta \cdot L^2}{I_o} \right) \quad (17)$$

where  $L$ ,  $B$  and  $T$  are the length, beam, and draft of the vessel. The aspect ratios  $B/T$  and  $L/B$  represent those of a plate constituting the projection of the hull surface normal to the direction of the motion. Equations 15 and 16 are based on added mass coefficients of a rectangular plate (4) modified to account for ship shapes by the incorporation of the coefficients  $C_M$ ,  $C_B$  and  $C_{wl}$ . Equation 17 is obtained from Ref. (36) based again on the flow around a flat plate in pitching motion.

The effects of the magnitude of the ramming speed, the bottom proximity, the distance floe edge or wall proximity, the free surface, and the frequency of the motion are represented by the coefficients  $K_{sp}$ ,  $K_{bp}$ ,  $K_{wp}$ ,  $K_{fs}$  and  $K_{fm}$  in the following

$$\lambda = \hat{\lambda} \cdot K_{sp} K_{bp} K_{wp} K_{fs} K_{fm} \quad (18)$$

where  $\hat{\lambda}$  is the basic added mass coefficient given by Eqs. 15 - 17 above.

Given existing knowledge (32, 36, 47) these coefficients are assumed to be given by

$$\begin{aligned}
 K_{sp} &= 1.0 \\
 K_{bp} &= 1 + \frac{0.4}{e/T} && \text{for heave and pitch} \\
 &&& \text{where } e = H - T \\
 &&& \text{(water depth-draught)} \\
 &= 1.0 \text{ for surge} \\
 K_{wp} &= 1 + 0.375 \left(1 - 3 \frac{x}{L}\right) && \text{for } 0 < x < L/3 \\
 &= 1.0 && \text{for } x > L/3 \\
 K_{fs} &= 1.0 \\
 K_{fm} &= 1.0
 \end{aligned} \tag{19}$$

which means that presently corrections can only be evaluated for bottom proximity  $e$  and distance to floe edge  $x$ . The water proximity  $e$  is equal to the water depth less the draught.

### 2.2.2 Flexural Vibrations

A number of analytical and semi empirical techniques are available for determining the added mass associated with small amplitude ship vibrations. These techniques are not however directly applicable to the impact simulation modelling. Since for the most part they deal only with open water conditions.

As an example of a semi empirical added mass coefficient Ref. 47 gives the following for the 2-node vertical mode:

$$\lambda = \frac{1}{3} \frac{B}{T} + 0.2 \tag{20}$$

where  $B$  and  $T$  are the beam and draft respectively.

The distribution of the added mass along the length of the hull can be calculated (48) using the expression

$$\Delta m_x = \frac{2}{3} \rho \frac{A_x b}{T_s} (1 + K L/B) \cdot \Delta x \quad (21)$$

where

$\Delta m_x$  is the incremental added mass in the segment of length  $\Delta x$  under consideration;  $\rho$  is water density;

$A_x$  is the submerged cross sectional area;

$b$  is the half width of the section;

$T_s$  is the draft of the section; and

$K$  is the empirical coefficient, dependent on the mode of flexural vibration;

$L, B$  are the length and breadth of the ship, respectively.

The value of  $K$  is taken as 0.1, 0.07 and 0.05 for the first, second, and third flexible modes, respectively. These coefficients apply to a range of vessels trading on rivers and inland waterways and therefore their applicability to icebreaking vessels, is to be confirmed.

The effect of bottom proximity can be expressed (43) via the correction factor  $\hat{K}_{bp}$  given in Table 1.

TABLE I  
Bottom Proximity Effect Factor  $K_{bp}$

| $\frac{B}{2e}$ | $2T/B$ |      |      |
|----------------|--------|------|------|
|                | 0      | 0.5  | 1.0  |
| 0              | 1.0    | 1.0  | 1.0  |
| 1              | 1.06   | 1.12 | 1.22 |
| 2              | 1.37   | 1.50 | 1.59 |
| 3              | 1.62   | 1.92 | 1.92 |



### 2.3 Full Scale Data Analysis

During the summer and fall of 1983 a comprehensive full scale testing program sponsored by Canadian Coast Guard Northern was carried out by Canadian Marine Drilling Ltd. Two icebreakers, Canmar Kigoriak and Canmar Robert LeMeur, were tested. Approximately 600 rams against heavy first year and multiyear ice were carried out. Full details of these tests and the data collected are given in (6, 13, 14). The data gave unique information about the global ice forces acting on the ship as well as the associated hull girder response.

Due to the fact that the ice forces could be estimated from measured hull bending stresses and since the accelerations were measured, it was believed that the data could yield useful information about the magnitude of the added mass.

Figure 5 shows a schematic of the procedure used to derive the vertical added mass distribution along the hull from the measured bending moments and accelerations. The details of this procedure are discussed below.

#### 2.3.1 Vertical Added Mass Distribution

The bending moment and shear force diagrams were determined from the normal and shear strain measurements by integration over the cross sectional area of each of the instrumented hull girder sections. The resultant distributed load acting over the length of the hull girder was calculated by differentiating the shear force

$$q(x) = \frac{dV(x)}{dx} = \frac{V_2 - V_1}{\Delta x} \quad (22)$$

where  $V(x)$  is the shear force distribution. This distributed load is the sum of the inertial loads and the change in hydrostatic loads. The inertial forces are then calculated as

$$q_i(x) = q(x) - q_h(x) \quad (23)$$

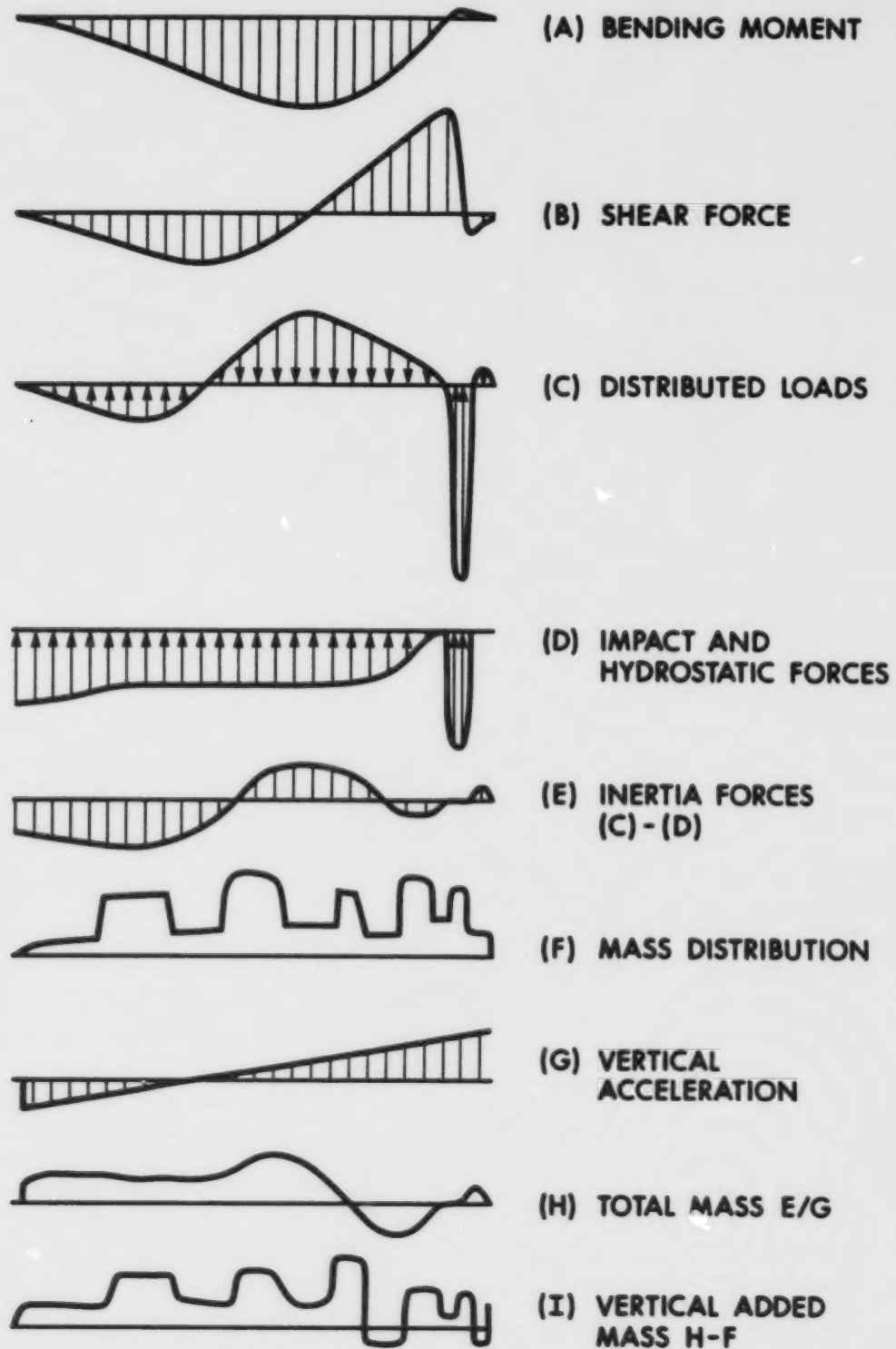


Fig. 5 Added Mass Calculation from Full Scale Data

where  $q_h(x)$  is the change in hydrostatic unit forces due to the motion of the ship. This change is not significant during the impact phase and is important in the beaching phase..

The accelerations  $\ddot{Z}(x)$  along the hull are calculated, Fig. 6, as follows:

- (a) Calculate the bow and stern vertical accelerations due to hull flexing (deflection).
- (b) Subtract the flexing accelerations from the measured bow and stern accelerations to obtain the rigid body acceleration components at the bow and the stern.
- (c) The acceleration at any point along the hull is then given as a rigid body component calculated using linear interpolation between the bow and stern values and the flexing accelerations calculated from the differentiation of the deflection twice with respect to time.

The unit added mass at any segment of unit length of the hull is calculated using the relationship

$$m_{az}(x) = \frac{q_i(x)}{\ddot{Z}(x)} - m_s(x) \quad (24)$$

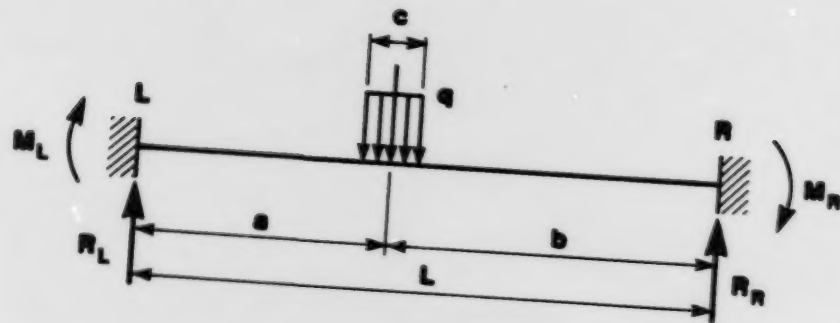
where  $m_s(x)$  is unit mass of the ship at location  $x$ .

### 2.3.2 Vertical Flexural Accelerations

The deflection of the hull girder was calculated using the method of elastic weights and conjugate beam analogy. In this method the bending moment is assumed to represent a loading condition on the conjugate beam of the hull girder which is a fixed-fixed beam representing the free-free hull girder. The bending moments due to this elastic loading condition are called the elastic bending moments and give the deflection of the beam due to the bending moment considered.



Fig. 6 Hull Girder Vertical Accelerations



$$M_L = \frac{-qc}{12L^2} [12ab^2 + c^2(L-3b)]$$

$$M_R = \frac{qc}{12L^2} [12a^2b + c^2(L-3a)]$$

$$R_R = \frac{qca}{L} + \left( \frac{M_L + M_R}{L} \right)$$

$$R_L = \frac{qcb}{L} - \left( \frac{M_L + M_R}{L} \right)$$

Fig. 7 Fixed-end Moments and Reactions

Figure 7 shows the equations used to calculate the fixed end moments and reactions due to a segment of uniform load of intensity  $q$  and length  $c$ . The load  $q_i$  on segment  $i$  of the beam is given by

$$q_i = \frac{\bar{M}_i}{EI} \quad (25)$$

where  $\bar{M}_i$  is the average bending moment in the segment.

The fixed end moments at the left and right ends of the beam are given by

$$M_L = \frac{-qc}{12L^2} (12ab^2 + c^2(L - 3b)) \quad (26a)$$

$$M_R = \frac{qc}{12L^2} (12a^2b + c^2(L - 3a))$$

The reactions are:

$$R_L = \frac{1}{L} (qcb - M_L - M_R) \quad (26b)$$

$$R_R = \frac{1}{L} (qca + M_L + M_R)$$

The deflection at any section along the hull is given by calculating the elastic bending moment at that section due to each segment of the beam and summing the results. For example the deflection at distance  $x$  astern due to the  $i$ th bending moment segment of width  $c$  shown in Fig. 7 is

$$z_{fi}(x) = R_L \cdot x + M_L \quad \text{for} \quad 0 < x \leq (a - \frac{c}{2}) \quad (27a)$$

$$z_{fi}(x) = R_R \cdot (L - x) - M_R \quad \text{for} \quad (a + \frac{c}{2}) \leq x \leq L \quad (27b)$$

The total deflection due to  $n$  bending moment segments is then given as

$$z_f(x) = \sum_{i=1}^n z_{fi}(x) \quad (27c)$$

These deflections are then differentiated twice to obtain the flexing accelerations which were used to compute the total acceleration distribution along the hull.

### 2.3.3 Hydrostatic Forces

The hydrostatic forces were determined by calculating the total vertical displacement at each section and assuming that the change in hydrostatic force is given as

$$q_h(x) \rightarrow \rho g B_x z \quad (28)$$

where  $q_h(x)$  is the change in the hydrostatic force at distance  $x$  due to a vertical motion of magnitude  $z$ .

The displacement  $z$  was obtained by the double integration of the measured bow and stern accelerations and assuming the displacement to be linearly distributed along the hull. This assumption is satisfactory due to the fact that the hull deflections were two orders of magnitude smaller than the rigid body displacements.

The above procedure yields a distribution of the vertical added mass due to the vertical rigid body modes, heave and pitch, as well as all the vertical flexible modes combined.

### 2.3.4 Added Mass Coefficients in Surge

The added mass coefficient in surge is calculated using the relationship

$$\lambda_x = \frac{F_x}{m_0 \ddot{x}} - 1 \quad (29)$$

where  $F_x$  is the horizontal bow force,  $\ddot{x}$  is the surge rigid body acceleration and  $m_0$  is the ship mass. Equation 29 gives an "effective" added mass coefficient since it includes the effects of damping and drag.

The force  $F_x$  is calculated using the relationship

$$F_x = \xi \cdot F_z \quad (30)$$

where  $\xi$  is the resolution factor given as

$$\xi = \frac{\mu + \tan \gamma}{1 - \mu \tan \gamma} \quad (31)$$

where  $\mu$  is the coefficient of friction and  $\gamma$  is the stem angle ( $\gamma = 24^\circ$  for the Kigoriak and  $30^\circ$  for the M.V. Arctic). The dependence of the resolution factor on the coefficient of friction is shown in Fig. 8 for the Kigoriak.



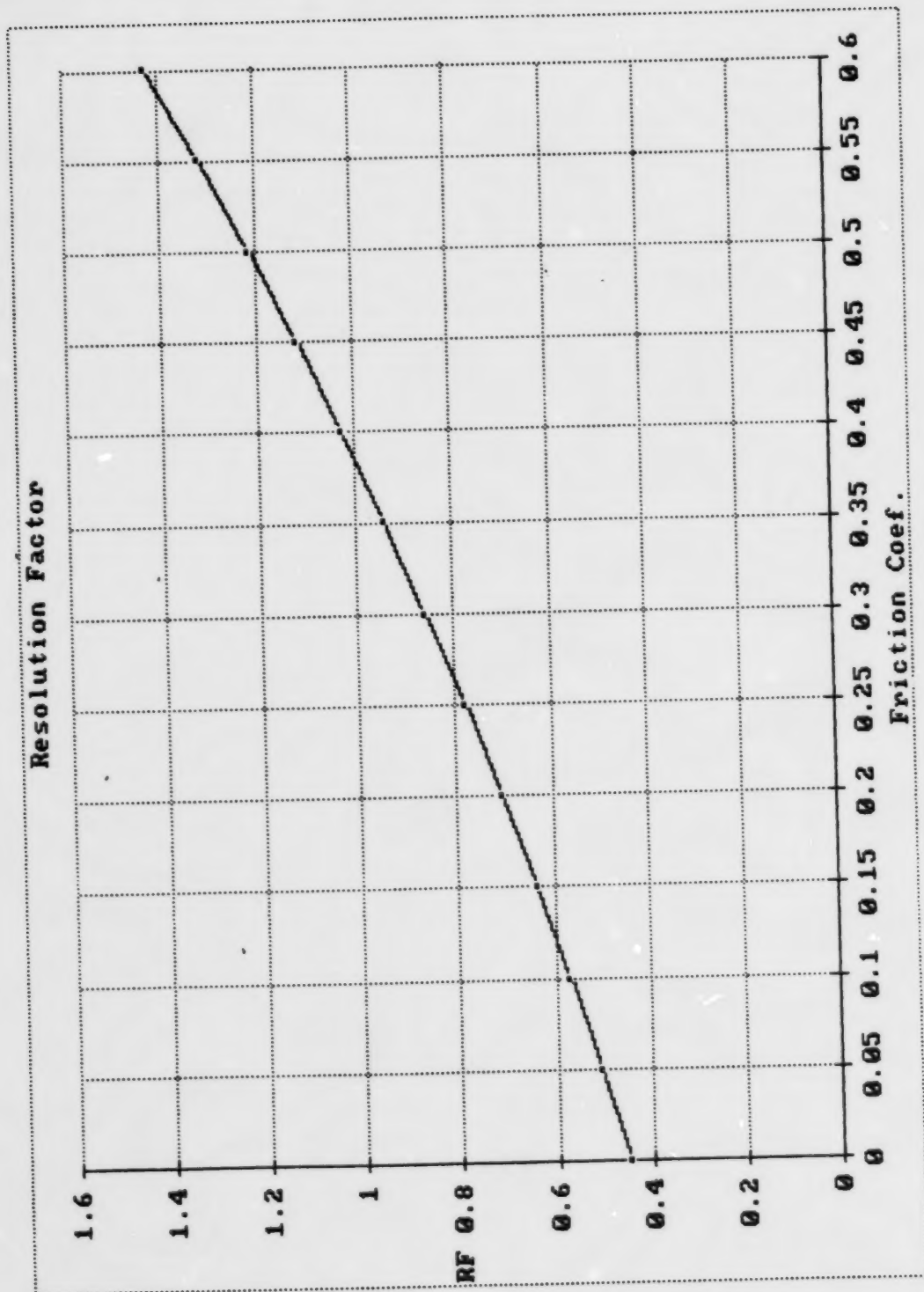


Fig. 8 Resolution Factor for Kigoriak

## **2.4 Model Testing**

A model testing program was designed utilising a 1:50 scale model of the M.V. Arctic and a 1:22.5 scale model of the Kigoriak. Offshore Research Ltd. (ORL) of Vancouver, B.C. was subcontracted to design a testing platform and an instrumentation system to carry out the tests at the National Research Council Institute for Marine Dynamics facilities, towing tank M-22, in Ottawa. The tank dimensions are 137 m x 7.6 m x 3 m and the towing carriage had a maximum speed of 12 m/s. The reports delivered by ORL are included in Appendix B of this report and give further details of the testing arrangement and details of the instrumentation system. This section presents an overview of the methods employed in designing the tests and in analysing the collected data.

### **2.4.1 Model Instrumentation**

Each model was instrumented for measuring the following (13 channels):

1. Wave height - capacitance wave probe.
2. Horizontal Impact Force - two component force dynamometer.
3. Vertical Impact Force - two component force dynamometer.
4. Velocity - carriage speedometer.
5. Bow vertical acceleration - vertical accelerometer.
6. Trim angle - angle transducer
7. Surge acceleration - Horizontal accelerometer.
8. Stern heave acceleration - Vertical accelerometer.
9. Heave displacement at C.G. - Ultrasonic heave transducer.
10. Hull pressure at bow - pressure transducer.
11. Midbody pressure - pressure transducer.
12. Midbody pressure - pressure transducer.
13. Stern pressure - pressure transducer.

#### 2.4.2 Data Acquisition System

The existing carriage based system was used for acquiring data. The system comprises:

1. DEC PDP-11/34.
2. HP 2648A interactive graphics terminal.
3. HP 2631G serial line printer with graphics.
4. DEC RK05F 5 megabyte fixed cartridge disk drive.
5. DEC RX01 dual floppy disk drive.
6. DEC KW11-K dual programmable realtime clock.
7. Neff 620-100 64 channel analog signal processor.
8. DEC IB-11 IEEE-488-1975 (GP1B).
9. HP 59309A HR1B digital clock.
10. Instrumentation cable.

Other equipment provided by I.M.D. for this program included:

Weighing scales  
Inertia table  
Video camera and recorder.

The following equipment was designed, fabricated and installed for the tests:

1. Adjustable false bottom, suspended from four winches so that it could be raised and lowered from the water surface to the tank bottom.
2. Ramming platform, to span the tank at water level, suspended from the brake rails at the end of the tank. Incorporating adjustable mounting arrangement for impact dynamometer and simulated ice floes.
3. Two component impact force dynamometer with clamping arrangement for foam blocks of different crushing strengths.
4. Adjustable simulated ice floes to mount onto ramming platform.

5. Impulse apparatus to apply a single vertical impulse at the bow or horizontally at the C.G., together with a strain gauged load cell to determine impulse magnitude and duration.

#### **2.4.3 Software**

The general data acquisition program RECORD of the Institute of Marine Dynamics was used. This system allowed sampling of 32 channels maximum in either static or dynamic modes with selectable time intervals between from 0.01, to 1.0 secs.

An HP 2648A interactive graphics terminal and HP 2614 serial line printer allowed the data for each channel to be printed graphically as a time series strip chart. Program RECORD also provided the basic statistics for each channel.

At the end of each test day, the test data was stored on floppy discs. The data was later transferred to a VAX mainframe computer which was linked to a plotter which produced the hard copy plots.

#### **2.4.4 Test Program**

After the facility had been set up, the M.V. Arctic was prepared and tested for free motions, forced motions and ramming. This was then repeated for the M.V. Kigoriak with only minor changes to the set up.

A 1/50 scale model of the M.V. Arctic with removable bow section was loaned by I.M.D. Prior to test the model was outfitted with instrumentation weighed and ballasted to a representative longitudinal moment of inertia using the Gospodnetic pendulum frame.

The original 1/22.5 scale model of the Kigoriak was loaned by Canadian Marine Drilling Ltd. Prior to testing the model was outfitted with instrumentation, weighed and ballasted to a representative longitudinal moment of inertia.

Two type of tests were carried out: free motion tests, and forced motion tests.

(a) Free Motion Tests

For the free motion tests each model was set up so that it was floating freely under the I.M.D. SEAKEEPING FRAME. This arrangement provided freedom in heave, pitch and surge but restraint in yaw and sway. Roll angle was also restrained but coupled to sway.

As there was no natural restoring force, long coil springs were connected between the vertical posts in the model and the mid point of the seakeeping frame. Thus when a surge offset was applied to each model it would freely oscillate. The arrangement also imposed a very small heave restoring force due to the geometry.

The instrumentation of both models was similar except for heave measurement. For the M.V. Arctic an ultrasonic transmitter was mounted on the underside of the seakeeping frame, so that the transmitted beam was reflected from the inside bottom of the model to measure heave motions.

For the M.V. Kigoriak model, the ultrasonic transmitter was mounted on the deck of the model facing upwards. The beam was reflected off a horizontal plywood sheet mounted on the underside of the seakeeping frame, parallel to the water surface.

Other transducers mounted similarly in both models included:

- Vertical accelerometers at the F.P. and A.P.
- Horizontal surge accelerometer close to midships
- Accelerometer calibrated to measure pitch angle
- Yoyo potentiometer to measure surge between seakeeping frame and the model
- Pressure transducers on the underside of model on the centreline
- Video camera mounted on forward extension to carriage.

The free motions of each model were recorded to determine the effect of forward speed, bottom proximity, and floe thickness.

(b) Forced Motion Tests

The test arrangement was similar to that of the free motion tests with the addition of an arrangement to apply a known disturbing force in either pitch, heave or surge. The arrangement for free and forced motions tests is shown in the photographs of Figures 9 and 10.

The instrumentation was similar to the free motions with the addition of a tension load cell to measure the disturbing force and its duration.

Two types of forced motions were applied:

- (1) Single forced impulses, in surge, heave at C.G. and in pitch (vertical at bow), of differing time histories were applied to each model and their responses recorded to determine the effects of:

Forward speed  
Bottom proximity  
Floe thickness.

- (2) Ramming

The arrangement for the ramming tests was similar to that of the forced motions on the Kigoriak model except that the surge restoring springs were removed and a clamping system arranged to restrain the model prior to ramming was installed.

Additionally a two component dynamometer was placed in the adjustable ice floe platform model at water level on the centreline of the tank. This held the simulated ice blocks which the models rammed against. The arrangement is shown in the photographs of Figure 11.





Fig. 9 Test Arrangement for Free Motion Tests

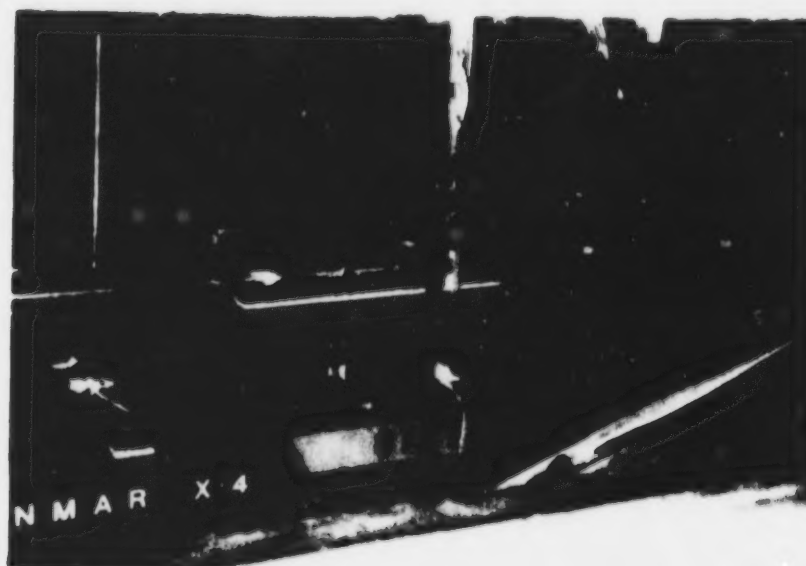


Fig. 10 Test Arrangement for Forced Motion Tests





Fig. 11 Ramming Test Arrangement

The instrumentation was similar to the Kigoriak free motion tests with the exception that the surge displacement yoyo transducer was removed. Also the video camera was removed from the carriage and mounted at the side of the tank, together with a capacitance wave probe to measure the reflected bow wave.

For each model the test series were similar with the model being towed at 5, 10 and 15 knots equivalent speeds to be released just prior to ramming the dynamometer containing simulated ice of various compressive strengths.

Ramming tests were repeated to determine the effect of:

- Forward speed
- Bottom proximity
- Floe thickness
- Floe compressive strength
- Floe surface friction.

Before leaving the facilities, the test data were transferred to the mainframe VAX computer. A copy was made on standard tape for subsequent analysis and the data for each instrumentation channel plotted out against time.

#### 2.4.5 Model Test Data Analysis

The following equations based on free oscillation theory of a single degree of freedom system were used in order to calculate the added mass and damping coefficients from the free oscillation tests; decrement curves analysis.

The equation of motion, Eq. 11a, may be simplified for a freely oscillating system as:

$$m_o (1 + \lambda) \ddot{x} + c\dot{x} + kx = 0$$

$$\lambda = \frac{k}{m_o \omega^2} - 1 \quad (32)$$

$$c = 2 \frac{\delta}{T_D} k m_o (1 + \lambda) \quad (33)$$

where  $k$  = stiffness and  $m_o$  is mass of model,  $\omega$  is the undamped circular frequency (rad/s),  $c$  is the damping coefficient and

$$\delta = \text{logarithmic decrement} = \ln (X_1/X_2) \quad (34)$$

the circular frequency is given as

$$\omega = \left( \frac{4\pi^2 + \delta^2}{T_D^2} \right)^{1/2} \quad (35)$$

$X_1$  and  $X_2$  are the motion amplitudes one period,  $T_D$  apart, Fig. 12. The damping ratio, defined as the ratio between the damping coefficient  $c$  and the critical value beyond which the system does not oscillate, is calculated using

$$\xi = \left( 1 + \left( \frac{2\pi}{\delta} \right)^2 \right)^{-1/2} \quad (36)$$

The stiffness  $k$  differs in the surge and heave directions. For surge there is no natural restoring force so the stiffness is equivalent to those of artificially attached spring which were required to induce the surge vibratory motions. The heave stiffness is equivalent to

$$k_h = \gamma_w A_{wl} \quad (37)$$

where  $\gamma_w$  is the unit weight of water and  $A_{wl}$  is the waterline area of the model.

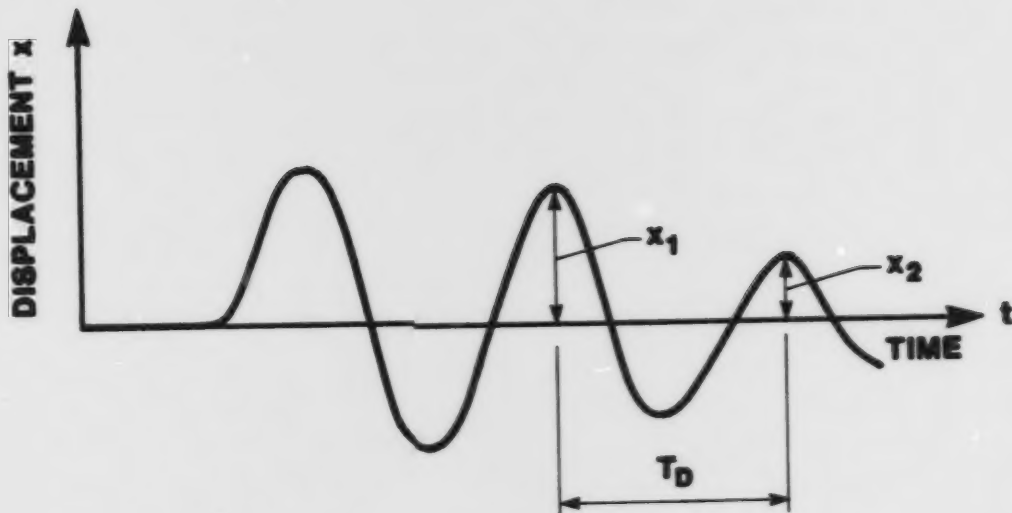


Fig. 12 Free Motion Signal

The values  $k$  and  $m_o$  in the above equations are replaced by  $K_\theta$  and  $I_o$  for the pitching motion where  $K_\theta$  is the angular stiffness and  $I_o$  is the pitching mass moment of inertia.

The difference between the impulse or force motion type tests and the ramming tests lies in the shape of the force time history, Figure 13. The effective added mass coefficients are calculated for the impulse and ramming tests using the relationships

$$\lambda_x = \frac{F_x}{m_o \ddot{x}} - 1 \quad (38)$$

$$\lambda_z = \frac{F_z - \rho g A_{wl} z_G}{m_o \ddot{z}_G} \quad (39)$$

$$\lambda_\theta = \frac{M - \rho g I_{wp} \theta}{I_o \ddot{\theta}} - 1 \quad (40)$$

where

$M = F_z L_b$  is the moment of the impact force about the C.G.,

$F_x$  and  $F_z$  are the impact horizontal and vertical forces,  $L_b$  is the distance from the location of  $F_z$  to the centroid,  $\ddot{x}$ ,  $\ddot{z}_G$ , and  $\ddot{\theta}$  are the surge, heave and pitch accelerations. The C.G. vertical acceleration and the pitching acceleration are calculated from the bow and stern vertical accelerations  $\ddot{z}_b$  and  $\ddot{z}_s$  as follows, Fig. 14

$$\ddot{\theta} = \frac{\ddot{z}_b - \ddot{z}_s}{L_{pp}} \quad (41)$$

$$\ddot{z}_G = \ddot{z}_s + \frac{(\ddot{z}_b - \ddot{z}_s) L_s}{L_{pp}} \quad (42)$$

$L_s$  is the distance of the stern accelerometer from the centroid, Fig. 14

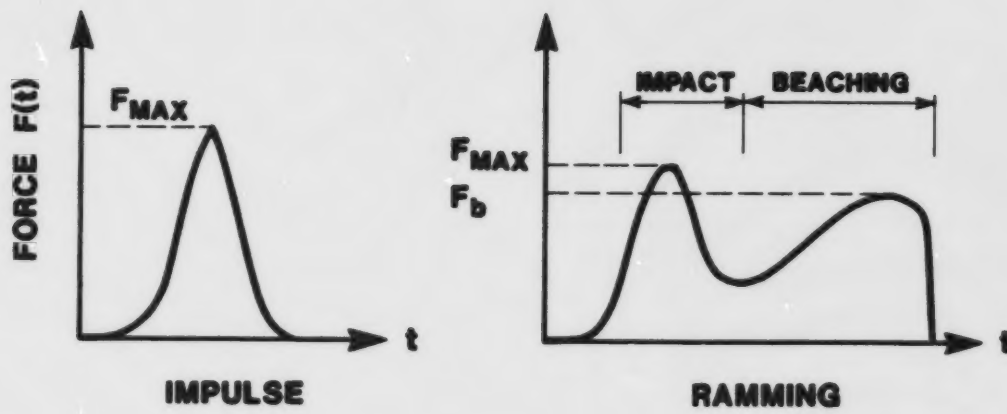


Fig. 13 Forcing Functions in Impulse and Ramming Tests

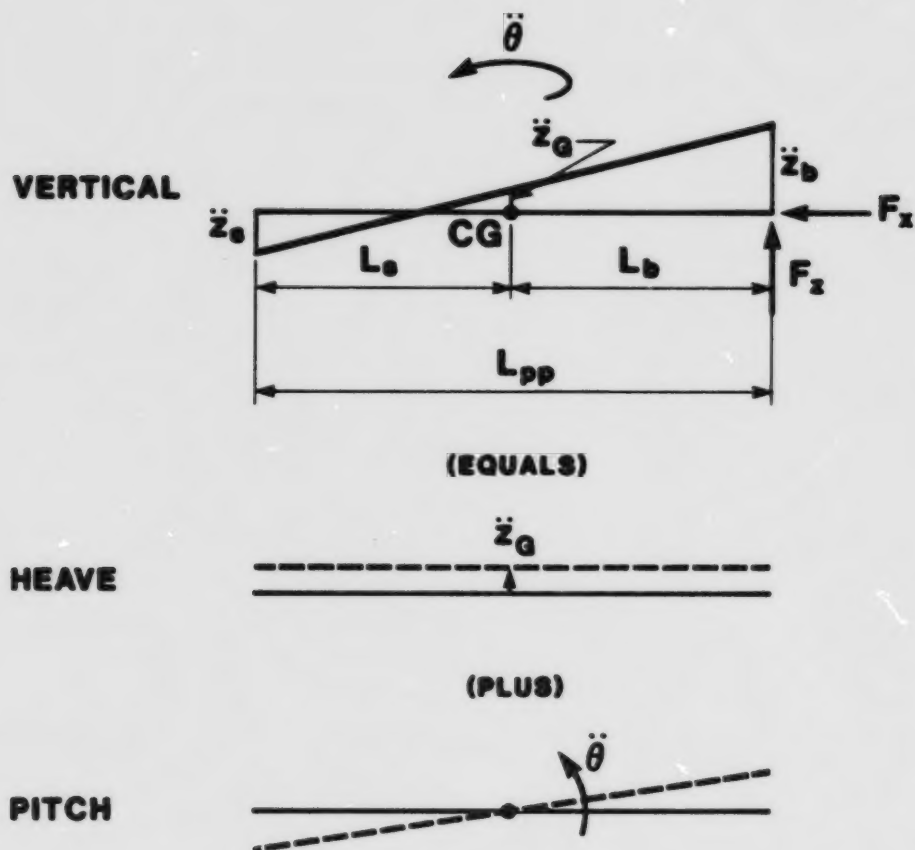


Fig. 14 Resolution of Vertical Accelerations into Heave and Pitch Accelerations



## 2.5 Finite Element Modelling

This section will discuss the theory employed in carrying out the finite element analysis and the ship, fluid and boundary idealization. The objective of the finite element work is to develop a numerical model capable of predicting the added mass of a ship of general hull form while ramming ice features of variable geometric properties and considering the effects of ice proximity and bottom proximity.

### 2.5.1 Theory

Part of the task of evaluating the hydrodynamic coefficients for a vessel during the various stages of collision with an ice feature involves using the finite element method to solve the linearised fluid potential flow around the vessel. In effect the fluid domain containing the vessel and the ice floe topography is prescribed to lie within finite boundaries, the fluid itself being subdivided into an assemblage of solid prismatic "bricks" or elements. The properties of the fluid as a continuum are faithfully retained in the idealisation as far as is reasonable. For example, it is assumed that the vibratory response of the vessel when it makes contact with the ice edge is of small amplitude, that is, small enough that second order convective accelerations of the fluid can be neglected. It is further assumed, in the approach adopted herein, that the fluid experiences inviscid, irrotational flow. In this manner the governing differential equation describing the fluid flow is the well known Laplace equation i.e.

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0 \quad \text{or} \quad \nabla^2 \phi = 0 \quad (43)$$

where  $\phi$  is the velocity potential.

This equation also describes steady state problems in heat conduction and is thus very useful should analogous solutions to a particular problem be sought. As will be seen later such an analogy between heat conduction and fluid flow has been made for a plane problem to allow the ABAQUS finite element package to be applied to

the added mass calculation.

In applying a numerical technique to the determination of the hydrodynamic problem one seeks in the first instance to quantify that body of fluid which appears to move with the vessel when it undergoes relatively high frequency rigid body or vibratory motion. In addition to establishing a single overall value of added mass associated with each of the body frequencies of interest, a detailed distribution of the "added" mass along the vessel is sought in the present context to enable the degree of influence of the presence of an ice floe on the fluid flow around the hull to be established. It might be assumed that the flow restriction caused by the floe presence produces only local modifications of the hydrodynamic pressure in which case one would be interested in determining where along the hull such an influence ceases to have an effect upon values of added mass calculated by "traditional" means. In other words, given that means exist for calculating appropriate quantities of added mass for individual ship cross sections for open water conditions, (e.g. strip theory, Ref. Lewis (56), Frank's Close-Fit, Ref. (57), what are the modifying influences introduced by the presence of the ice?

Thus the numerical idealisation besides describing the geometry of the sub surface portion of the vessel must also take account of the position and topology of the ice floe. With parallel model test program being carried out ideally the circumstances chosen to describe the model tests should be identical to those of the theoretical investigation in order to identify and quantify the applicability of this approach for future vessels.

A two dimensional idealization of the problem was considered to indicate the modifying influence on hydrodynamic hull pressures due to bottom proximity and wall effects from a restricted channel. These results had a bearing on the choice of the tank width of the test facility since the use of a relatively narrow flume, (of the order of 4 times the beam of the model) creates a poor representation of open water conditions. In the following a brief description of this two dimensional analysis is given followed by a description of the methodology of solving a coupled structural/fluid vibration problem in three dimensions. The means of idealizing the

vessels' stiffness properties and fluid properties are described and the advantages of using a superelement approach outlined.

#### 2.5.1.1 The Two Dimensional Case

Strip theory is a relatively well established means (Refs. 56, 57) of obtaining hydrodynamic "added mass" coefficients. In essence various sections along the length of the ship are assumed capable of "shearing" type movement, in which case the further assumption is made that the surrounding fluid flow is two dimensional in character. One knows that the actual motion of the fluid is not two dimensional, so a correction has to be made for flow in the lengthwise direction. This correction is made using the so called 'J' factor. Nevertheless, in a number of problems, for example determining the sensitivity of the hydrodynamic coefficients to bottom proximity effects the two dimensional approach is adequate and of course computationally cheaper than investigating coupled fluid structure interaction in three dimensions.

The finite element program ABAQUS was used to model the problem illustrated in Fig. 15 which is similar to the blockage or "squat" phenomenon which occurs when barges/ships attempt to navigate restricted waterways. Much has been written elsewhere (see for example Refs. 58, 59, 60 and 61) on the use of fluid finite elements so that it is considered superfluous to repeat the development of the relevant mathematics herein.

It suffices to note that the finite element discretisation of the fluid surrounding the hull does not require the restrictive hypotheses, such as two dimensional flow accompanying the classical approach followed, for example, by Lewis (Ref. 56). The connection between the water and the complex structural geometry of the ship form is relatively easy to obtain using elements having curved boundaries, that is, the so called "isoparametric elements". The application of the finite element method to any vibration problem results in the formation of stiffness, mass and if required, damping matrices, which are representative of these properties of the system. In the case of

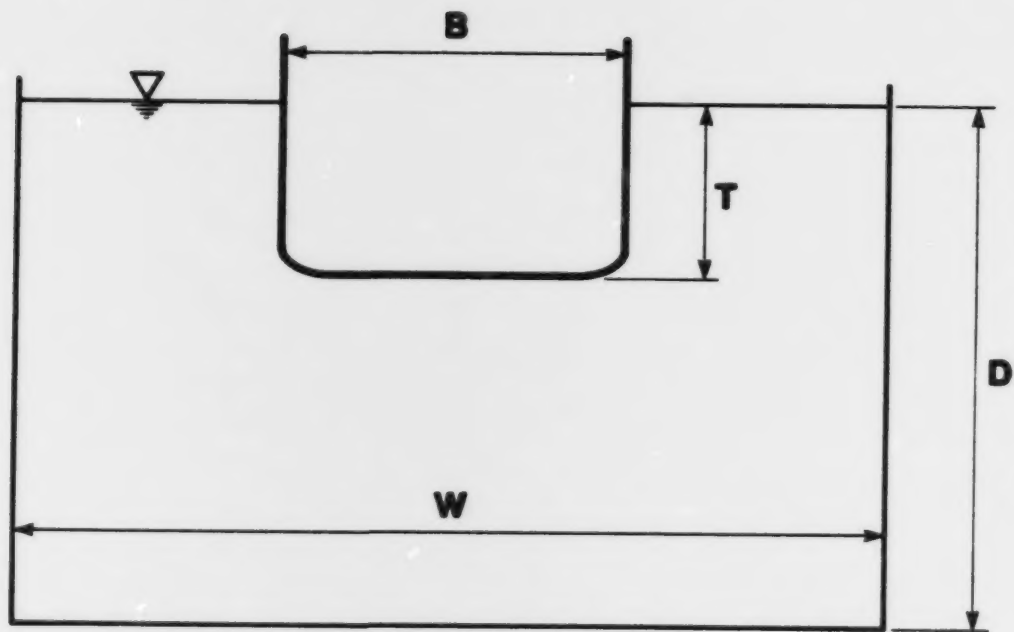


Fig. 15 A Body in a Restricted Waterway

ship vibrations, the mass of the system, as was noted earlier, is not simply that of the vessel itself but rather includes a representation of those portions of the surrounding fluid that are not set in motion as a result of the vessels vibratory movement.

We shall recall briefly the assumptions underlying the formation of the hydrodynamic mass matrix representing the discretisation of the disturbed fluid region,

- (a) incompressible, perfect fluid (i.e. no viscous effects)
- (b) irrotational flow
- (c) no surface waves (zero pressure at free surface)
- (d) no radiation potential at the infinite/far field.

The fluid equations of motion are thus

$$\nabla^2 p = 0 \quad \text{within the fluid domain} \quad (44)$$

$$\text{and } \frac{\partial p}{\partial n} = \rho \ddot{u}_N \quad \text{on that part of the hull surface in contact with the fluid.}$$

In words, the pressure gradient normal to the hull surface is directly proportional to the normal acceleration of that surface. Thus the sea bed and far field surfaces, assumed motionless, have a value of  $\partial p / \partial n = 0$ . In physical terms the pressure derivative is analogous to a flux exuding from a geometrically fixed surface. Thus the flux attempts to reach the free surface (zero pressure) by as short a route as is physically possible given the fluid properties.

As a first step, at defining the effects of rigid boundaries on the flow around a hull, the Kigoriak model was represented crudely by a rectangular section of width B at a working draft T (Fig. 15). It was placed in mid channel of a flume whose width dimension was assumed to be 4B in the first instance and 7B in a subsequent analysis. By virtue of symmetry only half the vessel/flume had to be modelled. Figure 16 illustrates the F.E. idealization that is 10 elements deep and 12 elements across the width. Each element has a mid side node enabling quadratic variation of pressure (temperature) along its edge i.e. a curved boundary can be followed. Note that the



1



term "node" when used in the context of a finite element idealisation of a surface or region is a joint or connecting point at which adjacent elements are assumed to equilibrate some physical quantity such as force or hydrodynamic pressure within the continuum being modelled. It thus has a totally different meaning from the same term used in vibration studies to denote a point on a particular mode shape which has zero movement in the appropriate direction. The vessel was assumed to undergo vertical (heave) translation so that unit flux was prescribed in the vertical direction at nodes NS1 -NS7 (nodes on the "bottom" edge of the vessel). Figure 17 shows the variation in hydrodynamic pressure on the box like section for three different bottom depths 2T, 3T and 10T (where T is vessel draft). Keeping the width of channel fixed at 4B. The results are meant to be descriptive and clearly demonstrate that as blockage increases (channel becomes shallower i.e. 2T) the hydrodynamic pressure increases.

The model also indicates that if hull accelerations are prescribed as flux then hydrodynamic pressure distributions can be obtained. The added "force" is simply the integral of such pressures over the surface Fig. 18, in the appropriate co-ordinate direction. These results indicated that it would be unwise to perform the model tests in a narrow flume such as that available at B.C. Research since condition (d) above, that is no radiation potential at the far field, is violated by the walls of the flume acting as reflective boundaries.



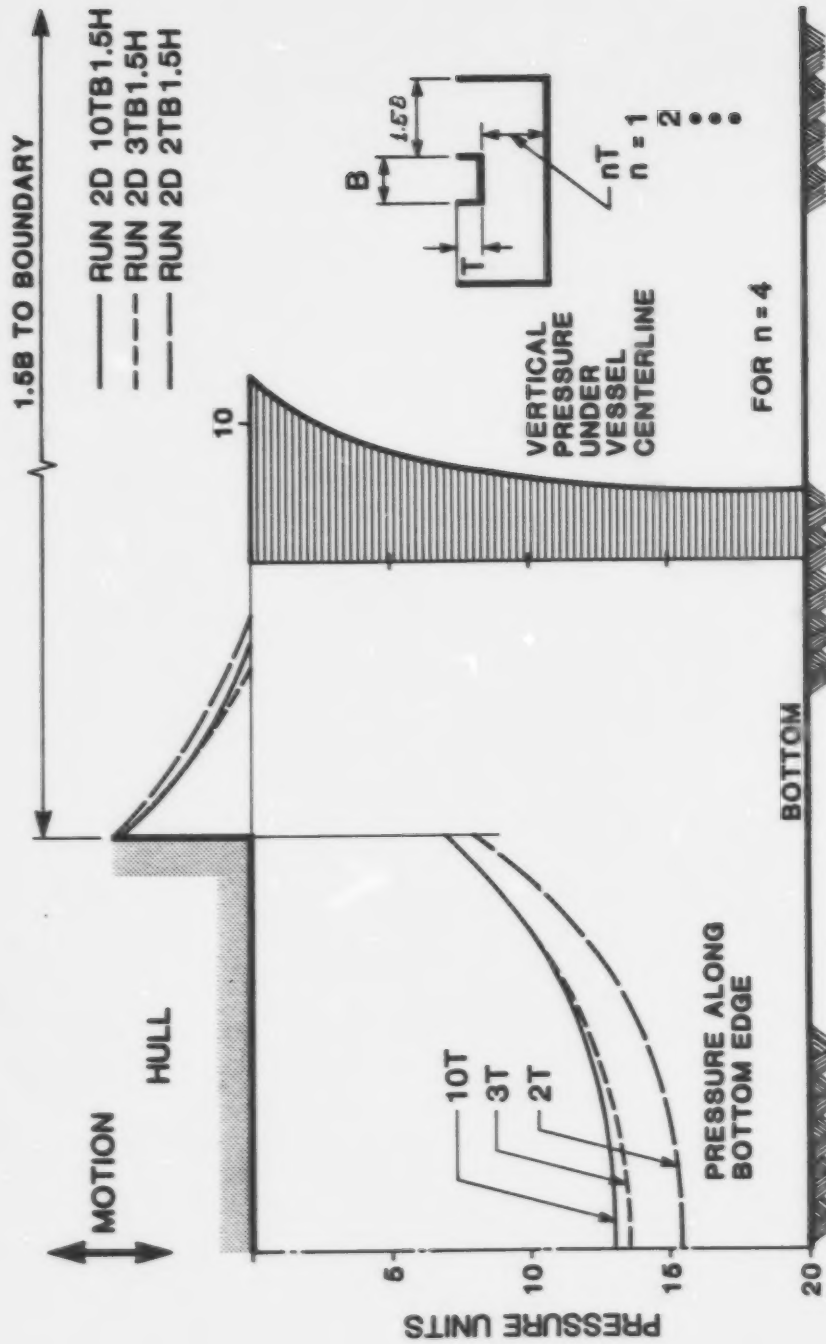
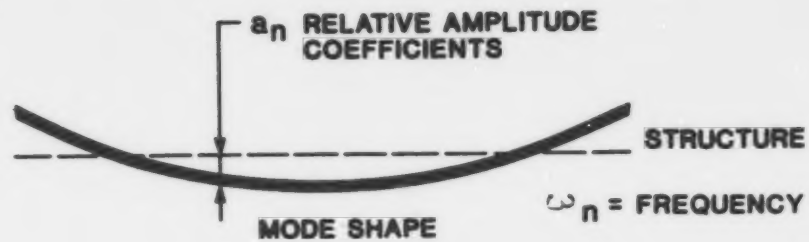


Fig. 17 Variation in Hydrodynamic Pressure Potential for Varying Bottom Proximity



RELATIVE AMPLITUDE COEFFICIENTS IN A PARTICULAR MODE CAN BE SCALED TO PROVIDE A DISTRIBUTION OF ACCELERATION ALONG LENGTH OF VESSEL

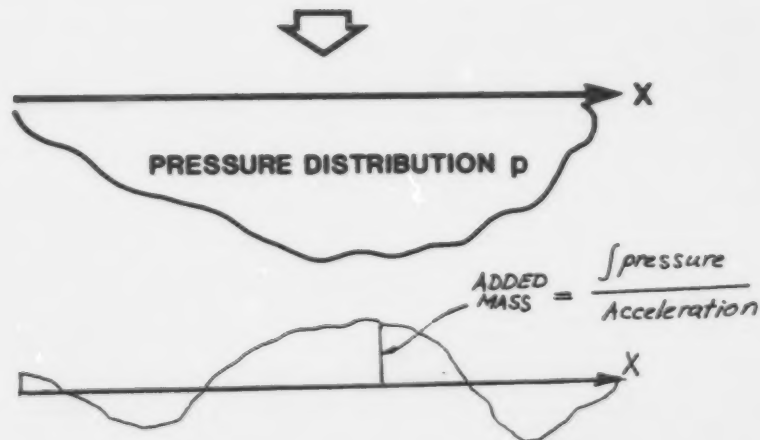
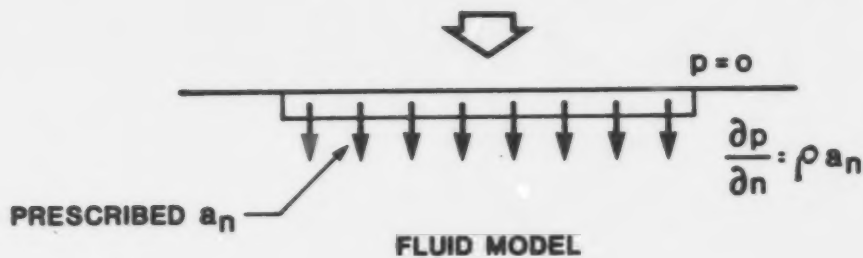


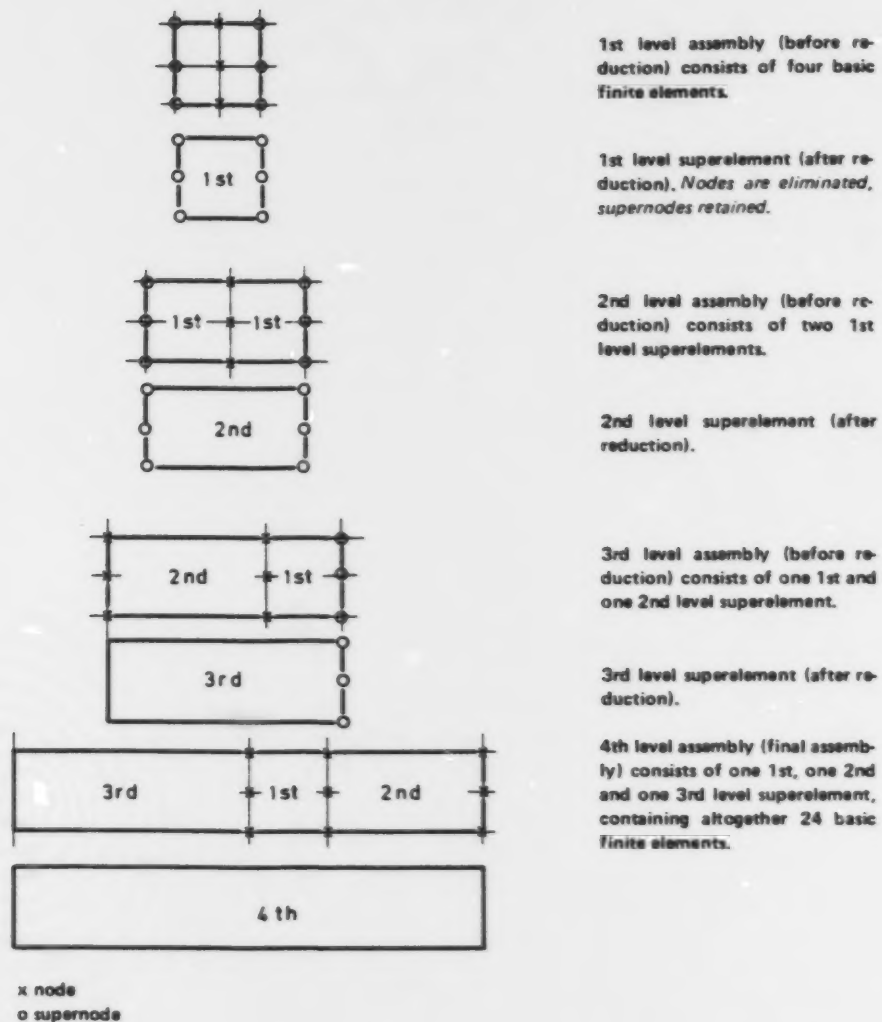
Fig. 18 Procedure for Determining Added Mass from Hydrodynamic Pressure Distribution

### 2.5.1.2 3-Dimensional Coupled Vibration Analysis

Since the added mass influences both natural frequencies and mode shapes it is appropriate that for each case wherein a change in added mass is foreseen that the natural frequencies and mode shapes are established. Ordinarily this would require an enormous amount of computation. The adoption of a superelement approach, available in SESAM 69 (DnV proprietary software) allows the problem to be made more tractable. Figure 19 illustrates the modular or "building brick" approach adopted in a general Superelement analysis. Individual pieces of structure or fluid are discretised and then "joined" to neighbouring superelements at boundary nodal points specially designated as "supernodes". Only these supernodes are retained at higher levels though the "reduction" process effectively ensures that the stiffness/mass properties of the individual elements are fully represented at the highest level. In the present circumstance we see the advantage of such an approach since the hull structure remains unaltered throughout the parametric investigation to be undertaken. Its stiffness and mass properties have only to be established once and held on file. Only the fluid superelements describing the floe geometry and surrounding fluid have to be altered for each separate run in the parametric study. Thus the amount of computation is kept to a minimum, it is still however quite formidable given the three dimensional nature of the idealisation.

As discussed earlier, for the coupled vibration problem both the hull flexibility equations and the flow equations have to be solved. The results of the analysis should yield as output the distribution of the hydrodynamic forces along the hull girder for each of the natural vibration modes of the coupled system.

The method employed to arrive at this result was to solve the coupled fluid-structure eigenvalue problem and back calculate the hydrodynamic forces. Results from a previous detailed three dimensional analysis of the dynamic response of the Kigoriak to ice ramming (35) indicated that the lower modes were essentially beam like in character. Accordingly, it seemed reasonable to represent the main hull girder as an elastic beam of variable flexural rigidity  $EI$ , shear stiffness  $GA_s$  and axial stiffness  $AE$ . The variable mass distribution of the lightship shown in



Note that supernodes specified in the assembly are retained in the superelement after reduction.

Fig. 19 Superelement Structuring in SESAM 69

Fig. 23 was also used in the representation. It was very important from a hydrodynamic viewpoint though that the three dimensional form of the hull was correctly represented so, rigid, massless frames defining the hull form were affixed to the flexible hull girder spine at main frame spacings (see Figure 20). Thus the surrounding fluid perceived a full form vessel and could transfer the hydrodynamic pressure acting on the hull surface to the main hull girder through the rigid frames. The frame rigidity enforced Bernoulli type bending response on the girder that is, plane sections remain plane during bending. This condition will tend to over estimate the hull stiffness slightly. Figure 20 also indicates the points of attachment of the buoyancy springs required to provide heave and pitch restoration, while a very light spring is shown affixed in the longitudinal direction at the aft most node point to prevent a singularity from occurring in surge. The buoyancy spring stiffness at each node was determined by calculating the force required to increase the draft of the element to which it is attached by one metre. That is by determining the localised tonnes per meter value. Strictly speaking the buoyancy springs should be attached to each transverse node point, to enable roll and coupled rolling motions to be represented correctly. However, the imposition of a vertical plane of symmetry along the hull girder centreline effectively reduced the rigid body modes to those of surge, heave and pitch making the transverse distinction of buoyancy an unnecessary complication.

The fluid is idealized using the assumptions of irrotational potential flow theory, care being taken to ensure that at the fluid structure interface each structural node had a corresponding fluid node for attachment. A superelement technique is employed within SESAM 69 in order to optimize the size of the model and the computational efficiency, its use is described in the following section.

Figures 21, 22 and 23 give the distribution of the Kigoriak equivalent beam properties; namely its moment of inertia, cross sectional area and mass utilized in the present analysis (solid line) compared with those used in a previous similar idealization (35). It should be noted that due to the assumed symmetry of the hull and the ramming scenario, it was considered adequate to model only one half of the structure and the fluid domain.

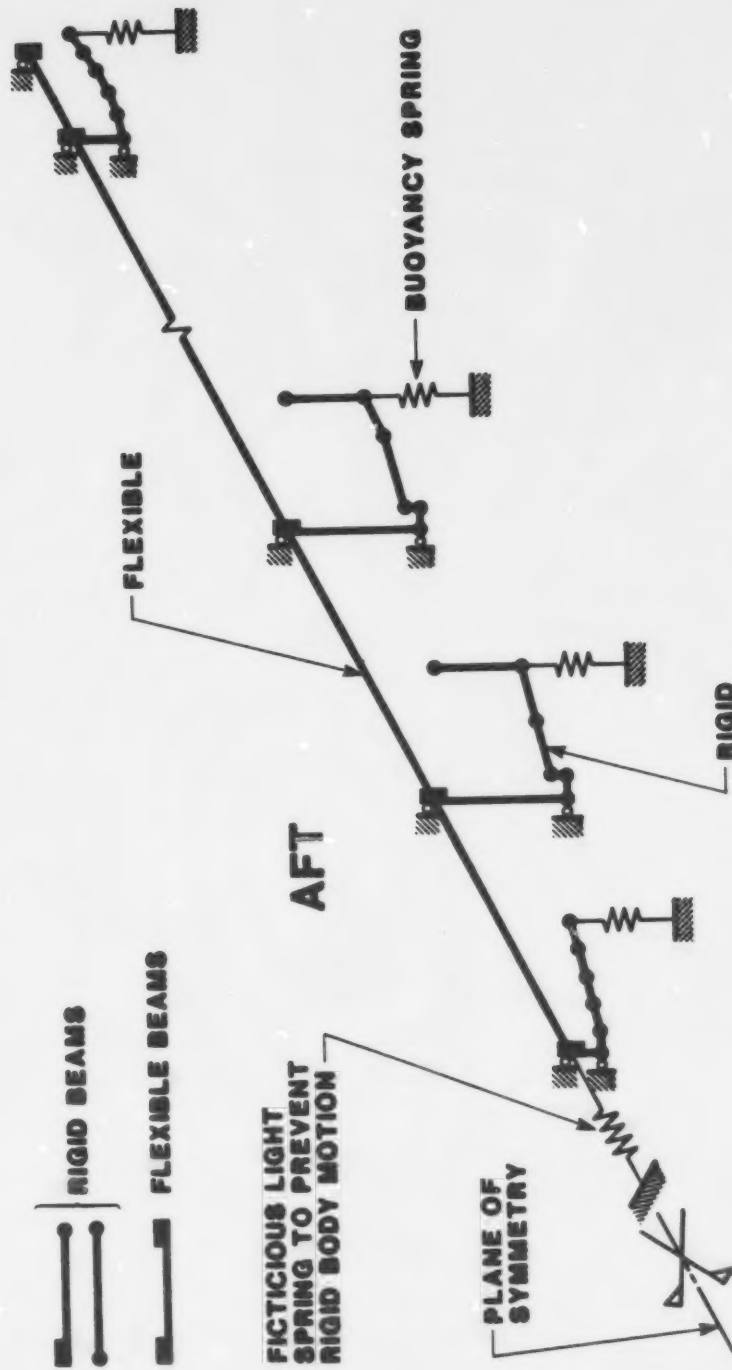


Fig. 20 Equivalent Beam Model of the Ship

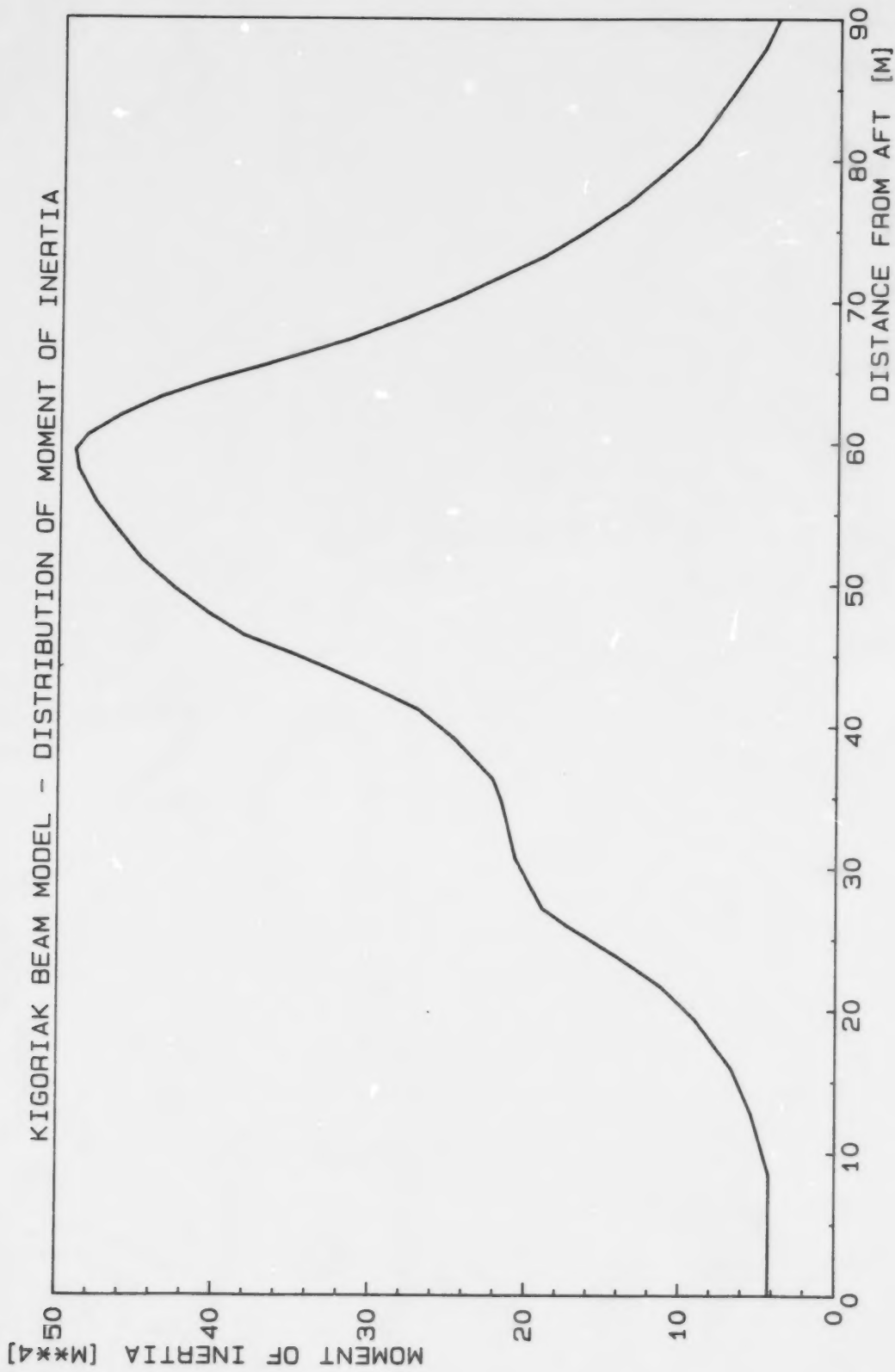


Fig. 21 Kigoriak Moment of Inertia Distribution



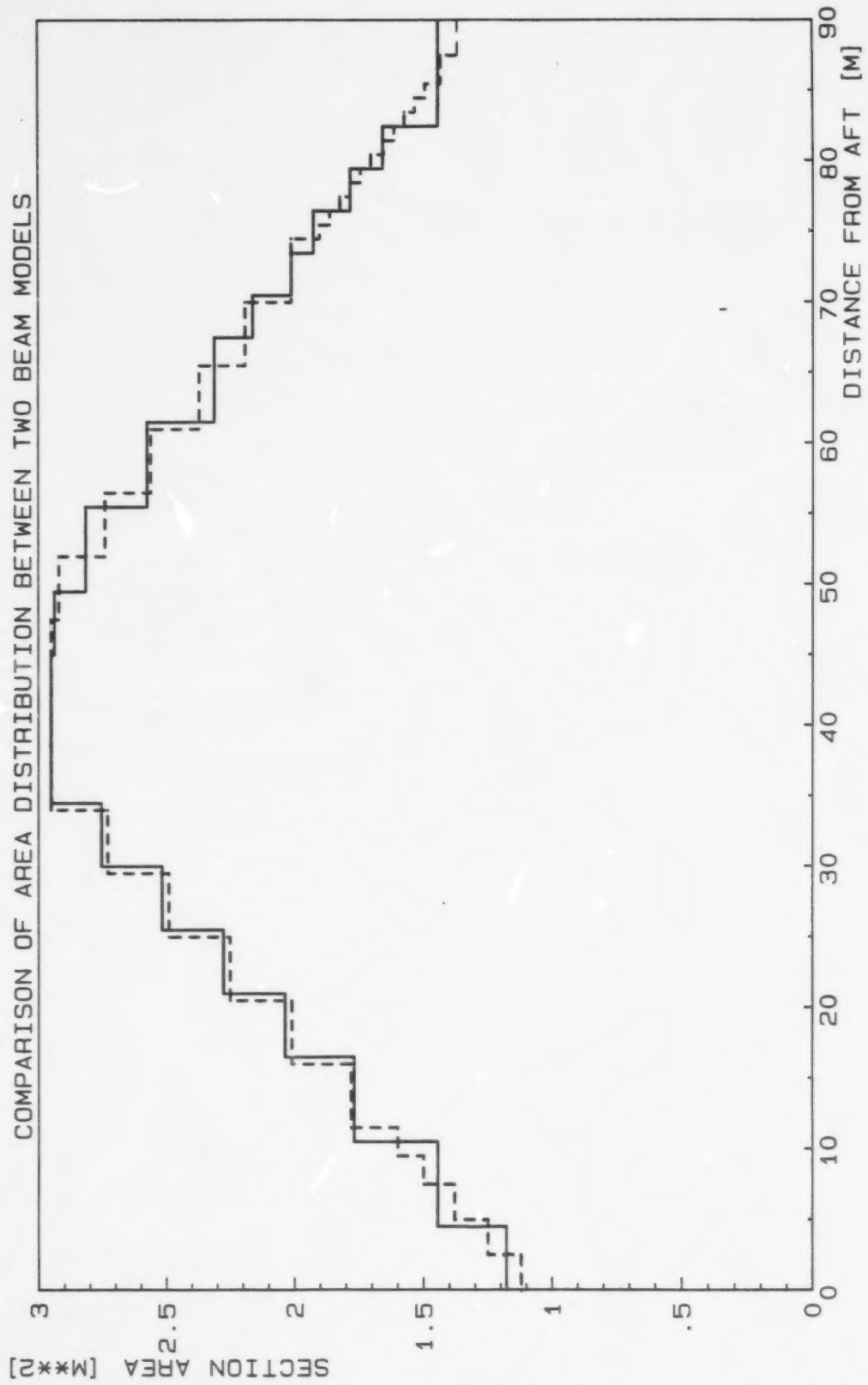


Fig. 22 Kigoriak Shear Cross Sectional Area Distribution

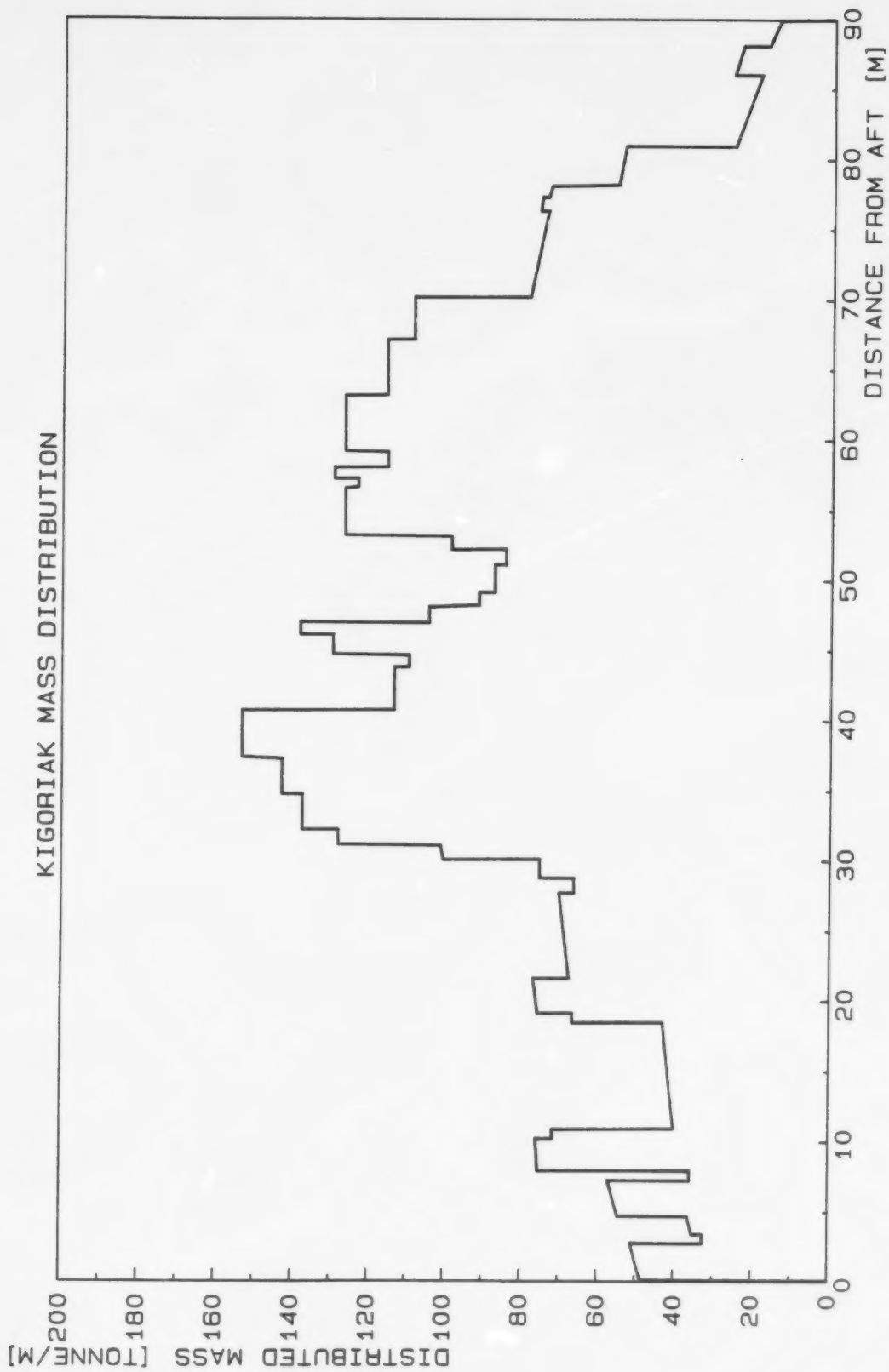


Fig. 23 Kigoriak Mass Distribution

The fluid domain was assumed to be incompressible and the flow was assumed to be irrotational validating the Laplacian equation for the hydrodynamic pressure  $p$

$$\nabla^2 p = 0 \quad (45)$$

with the boundary conditions:

- (a)  $p = 0$  at the free surface (46)
- (b)  $\frac{\partial p}{\partial n} = -\rho \ddot{u}_n$  with  $\ddot{u}_n = 0$  along ice boundaries, the sea floor and the far field  
and  $\ddot{u} \neq 0$  along the hull (the fluid/structure coupling condition)

It should be noted that a "node" is defined in FEM as a point where the degrees of freedom are unknown or the points where elements are connected. This is different from the definition of a "node" in vibration analysis as the points which do not translate while the hull is vibrating.

The fluid was modelled using 20-node brick type fluid elements. The nodal unknown values in the fluid are the hydrodynamic pressure  $p$ . These pressures were integrated along the hull surface and the resulting forces included in the dynamic equations of the structure/fluid system that is, assuming free oscillation, that is no external force applied,

$$M \ddot{X} + K X = F = -M_a \ddot{X} \quad (47)$$

or

$$(M + M_a) \ddot{X} + K X = 0 \quad (48)$$

The free vibration solution of Eq. 48 is

$$X = X_i e^{2\pi j f_i t} \quad (49)$$

where

$$\begin{aligned} X_i &= \text{eigenmode shape and} \\ f_i &= \text{eigenfrequency (Hz)} \\ j &= \sqrt{-1} \end{aligned}$$

Substituting (49) in (48) gives the generalized eigenvalue equation for the coupled problem:

$$K X_i - (2\pi f_i)^2 (M + M_a) X_i = 0 \quad (50)$$

Unfortunately, a computation difficulty, which was not foreseen at the project planning stage, arose when it became clear that in the complex file handling system adopted by SESAM 69, the hydrodynamic mass matrix was not explicitly accessible. Instead the combined structural and fluid mass was consolidated in the same matrix as shown algebraically in Equation 50. In order to separate the hydrodynamic component, the effort had to be expended to reformulate the problem for the vessel "in water" and "in the dry". It may be asked why if the mass of the lightship is known as in Figure 23 was this step necessary? The answer lies in the nature of the finite element modelling. A consistent rather than lumped mass approach has been used thus the mass matrix for the hull girder has a banded form, the off diagonal coefficients deriving from the structural coupling between the rotational and translational nodal degrees of freedom.

The hydrodynamic mass matrix is found at this point by subtracting the consistent structural mass matrix from the matrix representing the combined fluid and ship mass. As an additional check vibration frequencies for the "dry modes" were obtained and these when compared to the corresponding frequencies for the "wet" modes enabled an added mass coefficient to be obtained directly as follows

$$\omega_{\text{dry}} = \sqrt{\frac{\text{Stiffness}}{\text{Structural Mass}}} = \sqrt{\frac{K}{M}} \quad (51)$$

In order to avoid zero frequency rigid body modes in the dry analysis, buoyancy and surge restraining springs were affixed to the beam model. Thus, the stiffness of the system remained unaffected when the fluid elements were connected to the structure to determine the "wet" modes

$$\omega_{\text{wet}} = \sqrt{\frac{\text{Stiffness}}{(\text{Structural} + \text{Fluid}) \text{ Mass}}} = \sqrt{\frac{K}{M_W}} = \sqrt{\frac{K}{M(1 + \lambda)}} \quad (52)$$

where  $\lambda$  is the added mass coefficient for each mode in turn.

As stated previously the distribution of added mass along the length of the vessel is of substantial interest in addition to a lumped mass coefficient.

It is clear that the hydrodynamic mass matrix is given by the algebraic difference between the consolidated mass matrices computed for the wet and dry runs that is

$$M_A = M_W - M \quad (53)$$

The distribution of hydrodynamic force along the hull for in frequency mode  $i$  is thus given by,

$$F = (2\pi f_i)^2 M_A X_i \quad (54)$$

Hence once the frequencies and mode shapes are found from the eigenvalue analyses the hydrodynamic force distribution can be found.

### 2.5.2 Finite Element Mesh

The skeletal type model of the vessel is made up of beam elements each of which has six degrees of freedom per node. Since the fluid elements only have degrees of freedom in three orthogonal directions the nodal rotations on the skeletal frame have to be reduced out before "mating" the structure to the fluid. This reduction of the nodal rotations creates a second level superelement out of the assembly of the first level beam elements.

The mesh of the fluid at the lowest level consists of (20 node bricks), finite elements and represents the shape of the hull below the waterline as closely as possible. In addition, the choice of mesh size in the ice zone was such as to ease the modelling of the variation of proximity and thickness of the ice features. In other words to capitalise on the superelement approach it is desirable to reuse previously formed superelements in several analyses. The fluid is contained within a rectangular box whose dimensions were chosen to represent the field condition. The fluid has been discretized with 24 elements in the X direction (SURGE), 8 elements in the Y direction (SWAY) and 6 elements in the Z direction (HEAVE), accounting for a total of 1152 20-node brick elements, for the half model (symmetry across XZ plane).

A set of cross-sections through the mesh are shown on Figure 24 (YZ plane). These sections define the shape of the hull below water line.

A schematic elevation of the mesh (Fig. 25) shows basic finite elements forming first level superelements and defining the boundary of the hull.

Figure 26 shows the various boundary conditions examined.

After successive reductions (Fig. 27) of internal nodes up to the third level and prescription of free surface conditions, all the fluid degrees of freedom are reduced to only those at the surface of the hull (Fig. 28). The beam model is connected to the fluid at level 4 and this assembly is further reduced to include only the flexible degrees of freedom of the hull girder before the final eigen mode extraction takes place at level 5.

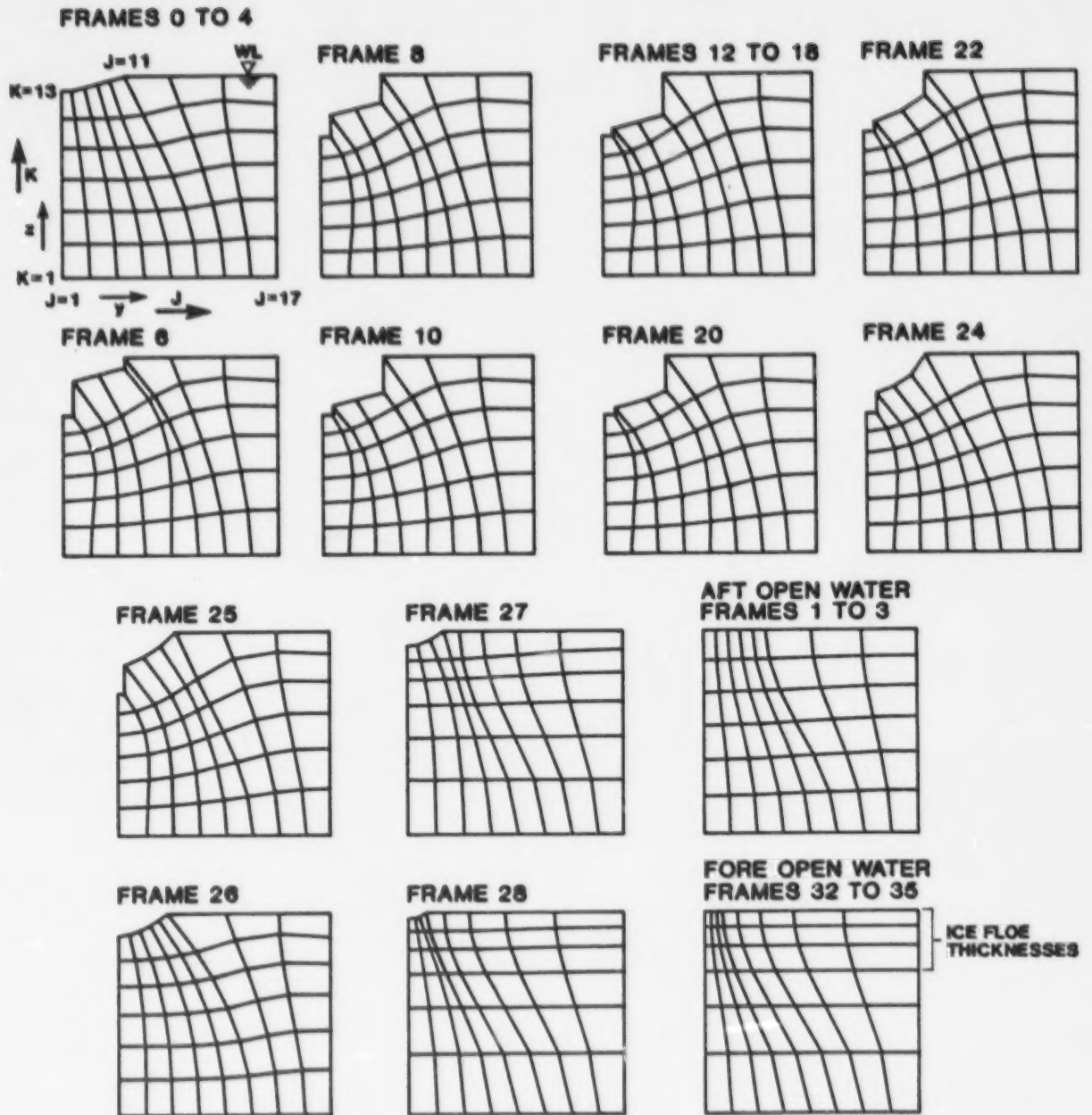


Fig. 24 Fluid Idealization



# FIRST LEVEL FLUID MODEL

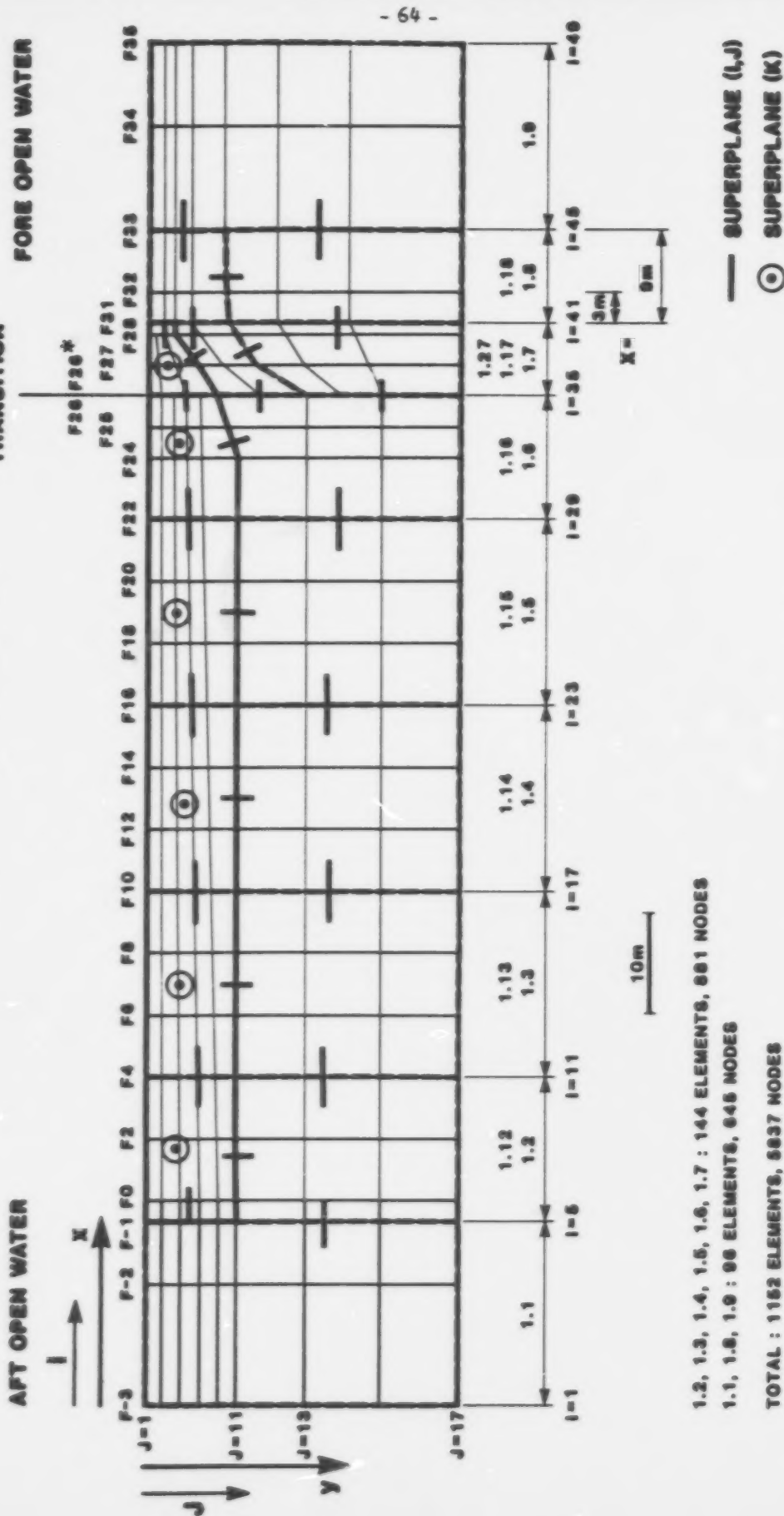


Fig. 25 First Level Superelements and Hull Boundary

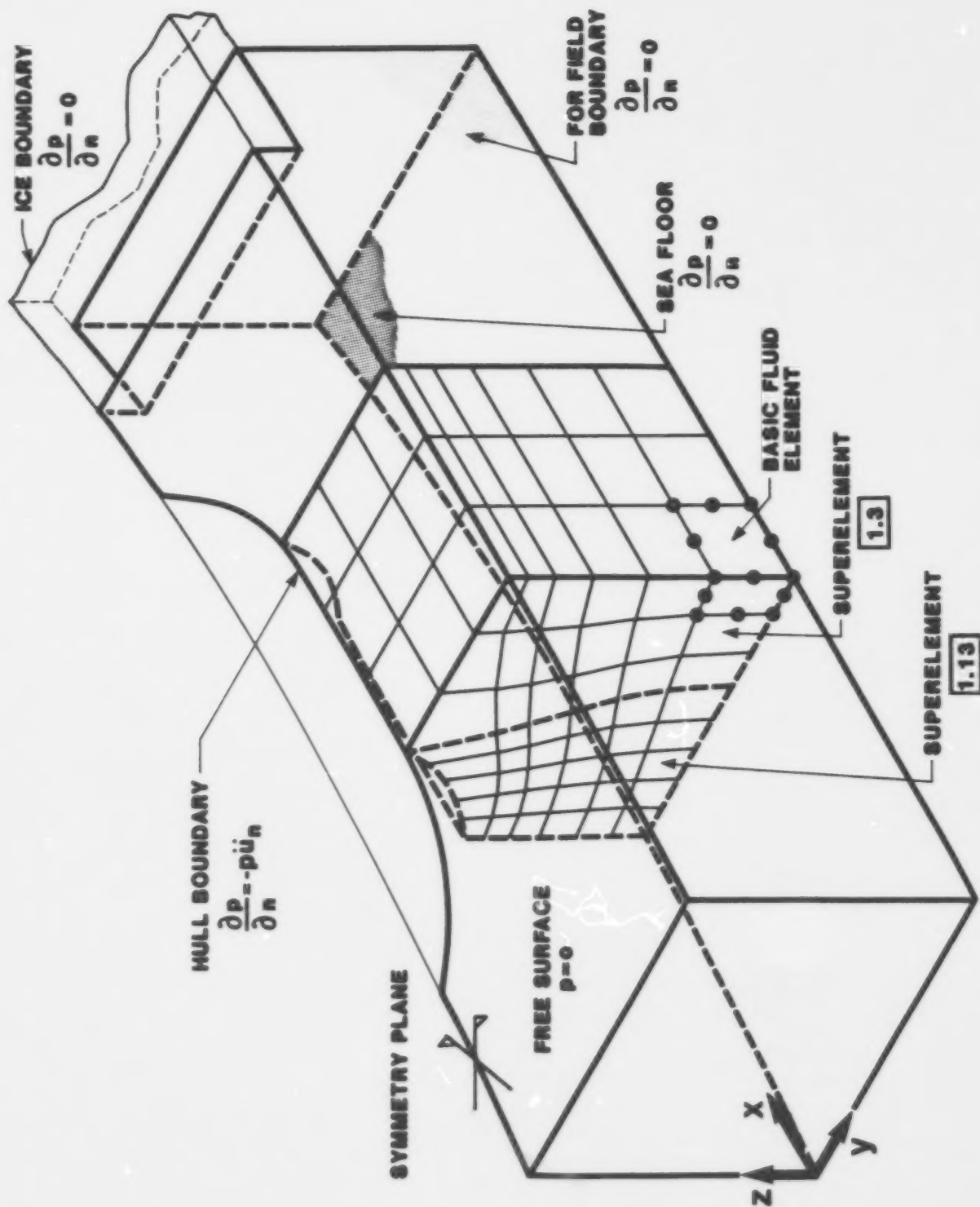


Fig. 26 First Level Superelements and Boundary Conditions

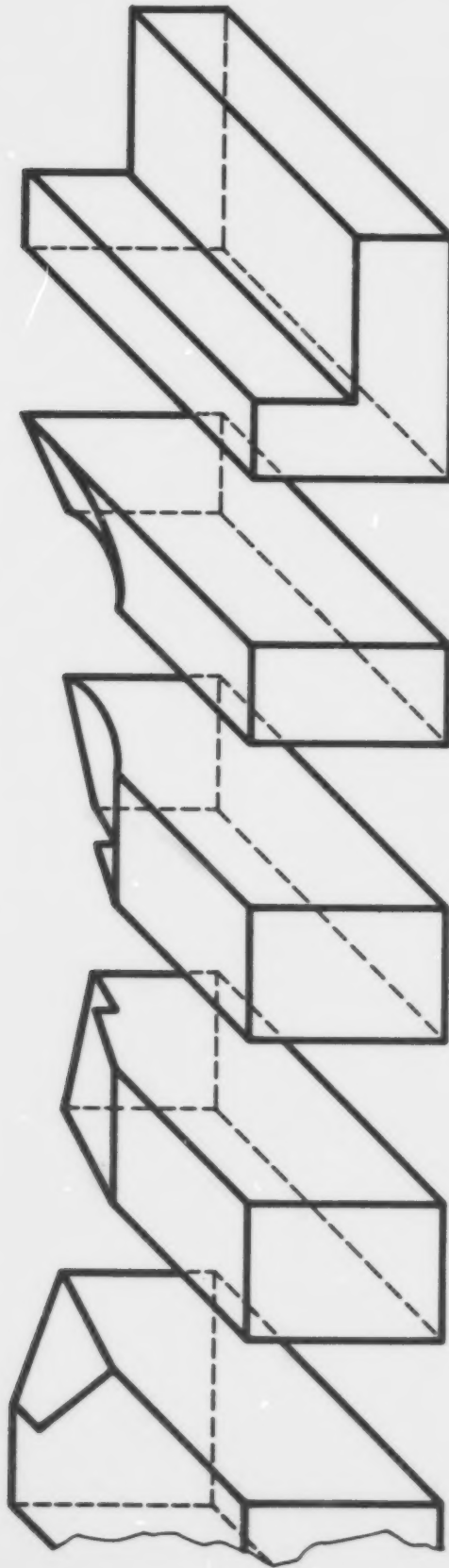
2.1

2.2

2.3

2.4

2.5



VARIABLE ICE BOUNDARY

Fig. 27 Second Level Superelements

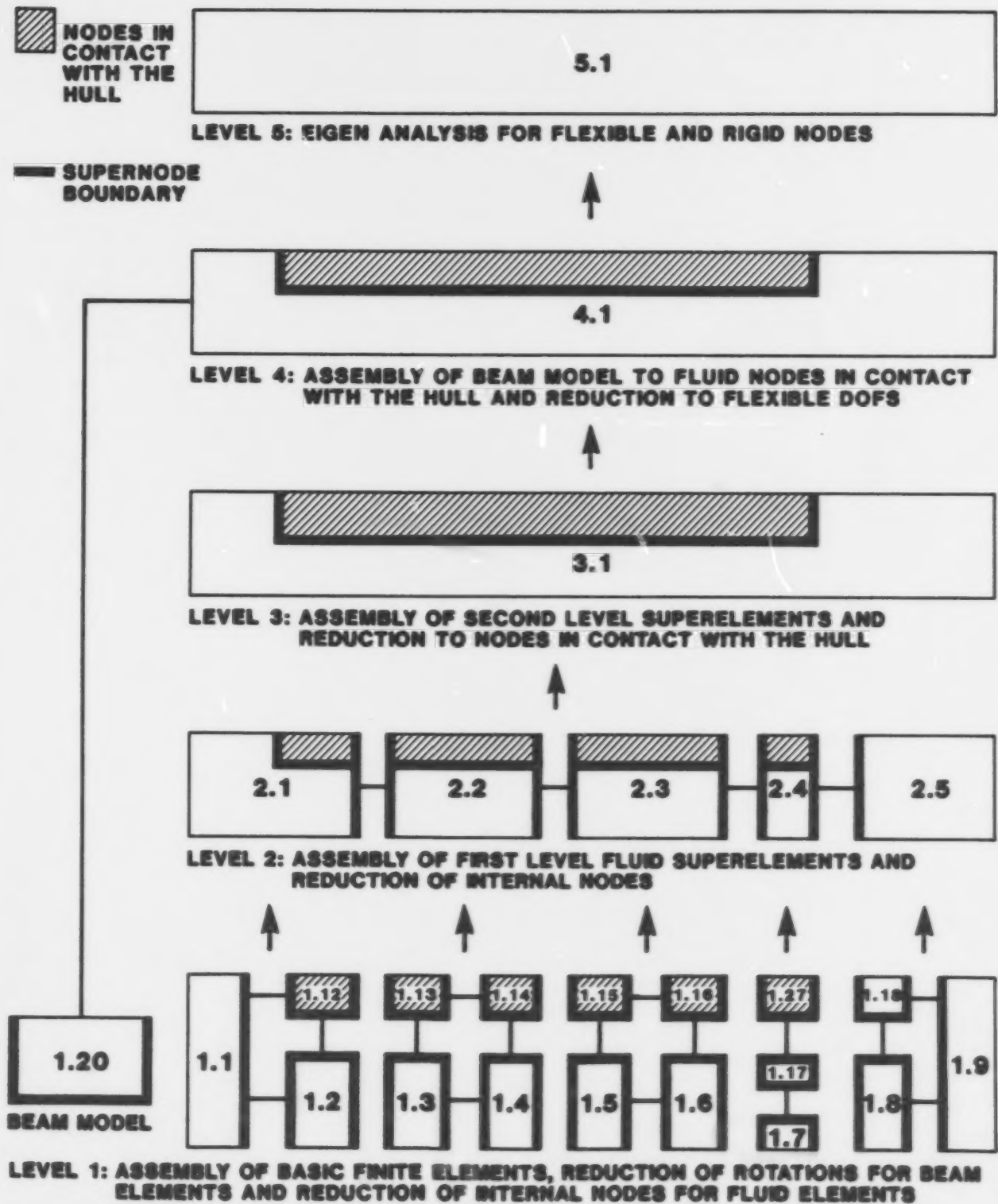


Fig. 28 Levels of Finite Element Superelement Model

## 2.6 Summary

This section presented the methodologies employed in the four approaches used to determine the added mass namely, analytical modelling, full scale data analysis, model testing and finite element modelling.

The state of the art formulas employed in calculating the added mass were presented together with proposed formulations based on two dimensional flow theories and corrections to account for such effects as bottom and ice proximities.

The methods employed in the analysis of the Kigoriak 1983 impact test data were described. The developed equations were programmed into the existing HPL version of the full scale data analysis program.

A comprehensive description was presented of the model testing program carried out using models of the M.V. Arctic and M.V. Kigoriak at the I.M.D. facilities in Ottawa. The methods employed to analyse the data were discussed in detail.

Finally the theories employed in the finite element model developed for the M.V. Kigoriak were outlined and a detailed presentation of the finite element mesh employed to idealize both the fluid and the hull structure and form were presented.

### 3.0 RESULTS

#### 3.1 Introduction

In Section 2.0 the theoretical background and the developed methodologies were presented. This section presents the results obtained from the work carried out under all tasks. The presentation of results given in Sections 3.2 to 3.5 gives salient results pertaining to the analytical modelling, full scale data analysis, model testing, and finite element analysis. Section 3.6 compares the results obtained from the various formulations and a summary of the results is given in Section of 3.7.

It should be mentioned that the data base available for analysis is comprised of 400 rams of the Kigoriak in heavy first year and multi-year ice and 275 model test runs of the M.V. Arctic and the M.V. Kigoriak. Full analysis of this data would require significantly more resources than presently available within the present scope. Therefore only sample representative data were analysed herein. This analysis is by no means exhaustive, that is, there is certainly considerable room for more extensive analyses to be carried out on this extensive data base.

Due to the relatively large number of figures associated with this Section, the figures for each subsection will be placed at the end of the text of that subsection.

##### 3.1.1 Particulars of the Kigoriak and the M.V. Arctic

Tables 2 and 3 entail significant parameters of the two vessels Canmar Kigoriak and M.V. Arctic and their tested models. This ship data was employed in the analysis of the full scale and model test data as well as in the application of the proposed analytical models.

Figures 21, 22 and 23 give the longitudinal distributions of the structural moment of inertia, cross sectional area, and mass distribution respectively for the Kigoriak.

TABLE 2  
M.V. CANMAR KIGORIAK MAIN CHARACTERISTICS

| Prototype |                                              | Model<br>(Scale 1:22.5) |                       |
|-----------|----------------------------------------------|-------------------------|-----------------------|
| Parameter | Value                                        | Parameter               | Value                 |
| LBP       | 79.3 m                                       | $L_{pp}$                | 3.52 m                |
| LWL       | 84.2 m                                       | $L_s$                   | 1.807 m               |
| LOA       | 90.7 m                                       | $L_b$                   | 1.713 m               |
| M.B       | 17.25 m                                      | $K_{sm}$                | 2,374 N/m             |
| D         | 9.0 m                                        | $K_{zm}$                | 26,740 N/m            |
| T         | 8.5 m                                        | $K_{\theta m}$          | 30,350 N.m/rad        |
| $\Delta$  | 7,000 TONNES                                 | $M_o$                   | 597 kg                |
| CB        | 0.59                                         | $I_o$                   | 595 kg.m <sup>2</sup> |
| $C_{wl}$  | 0.825                                        |                         |                       |
| L/T       | 9.33                                         |                         |                       |
| B/T       | 2.03                                         |                         |                       |
| B/L       | 0.22                                         |                         |                       |
| $C_{wl}$  | 0.88                                         |                         |                       |
| $C_M$     | 0.95                                         |                         |                       |
| $\gamma$  | 24°                                          |                         |                       |
| $I_o$     | 3.43 x 10 <sup>6</sup> tonnes m <sup>2</sup> |                         |                       |



TABLE 3  
M.V. ARCTIC (BEFORE REFIT) MAIN CHARACTERISTICS

| Prototype         |                                            | Model<br>(Scale 1:50) |                       |
|-------------------|--------------------------------------------|-----------------------|-----------------------|
| Parameter         | Value                                      | Parameter             | Value                 |
| LOA               | 209.55 m                                   | $L_{pp}$              | 4.11 m                |
| LBP               | 196.58 m                                   | $L_s$                 | 2.055 m               |
| M.B               | 22.93 m                                    | $L_b$                 | 2.055 m               |
| M.T               | 10.97 m                                    | $K_{sm}$              | 1,187 N/m             |
| SUMMER T          | 10.82 m                                    | $K_{zm}$              | 17,000 N/m            |
| $\Delta$ AT 10.82 | 37,724.1 TONNES                            | $K_{\theta m}$        | 20,455 N.m/rad        |
| CB                | 0.753                                      | $M_o$                 | 302 Kg                |
| CP                | 0.761                                      | $I_o$                 | 304 Kg.m <sup>2</sup> |
| TRIAL SPEED       | 15.5 KNOTS                                 |                       |                       |
| OPERATIONAL SPEED | 15.0 KNOTS                                 |                       |                       |
| L/T               | 18.17                                      |                       |                       |
| B/T               | 2.12                                       |                       |                       |
| B/L               | 0.12                                       |                       |                       |
| $C_{wl}$          | 0.87                                       |                       |                       |
| CM                | 0.99                                       |                       |                       |
| $\gamma$          | 30°                                        |                       |                       |
| $I_o$             | 95 x 10 <sup>6</sup> tonnes m <sup>2</sup> |                       |                       |

### 3.2 Analytical Model Results

The formulations presented in Section 2.2 were applied to the M.V. Kigoriak and the M.V. Arctic and the results are given in Table 4. The table shows the results for a rectangular barge pontoon for comparison purposes. The results show the basic coefficients before application of the correction factors which take account of such effects as bottom proximity, wall proximity, speed, free surface effects, and forcing frequency.

At present, it is estimated that for the impact scenario, the most significant effects are due to bottom proximity and wall (ice edge) proximity. The correction factors due to these two effects are plotted in Figures 29 and 30.

TABLE 4  
Calculated Added Mass Coefficients

| Ship        | Plate Formulation<br>Eqs. 15, 16 and 17 |             |                  | Literature*<br>Eqs. 12, 13 and 14 |             |                  |
|-------------|-----------------------------------------|-------------|------------------|-----------------------------------|-------------|------------------|
|             | Surge                                   | Heave       | Pitch            | Surge                             | Heave       | Pitch            |
|             | $\lambda_x$                             | $\lambda_z$ | $\lambda_\theta$ | $\lambda_x$                       | $\lambda_z$ | $\lambda_\theta$ |
| Kigoriak    | 0.13                                    | 1.1         | 1.042            | 0.075                             | 1.07        | 0.91             |
| M.V. Arctic | 0.05                                    | 0.91        | 0.80             | 0.075                             | 0.81        | 0.77             |
| Barge       | 0.04                                    | 1.61        | 1.18             | 0.12 - 0.18                       | 1.3 - 1.6   | 0.81 - 0.89      |

\* Ref. (36) for Kigoriak and M.V. Arctic and (54) for the Pontoon Barge

Applying Eq. 21 for the Kigoriak, an added mass density of 71.2 tonnes/m is obtained for the first flexural mode of hull vibration.

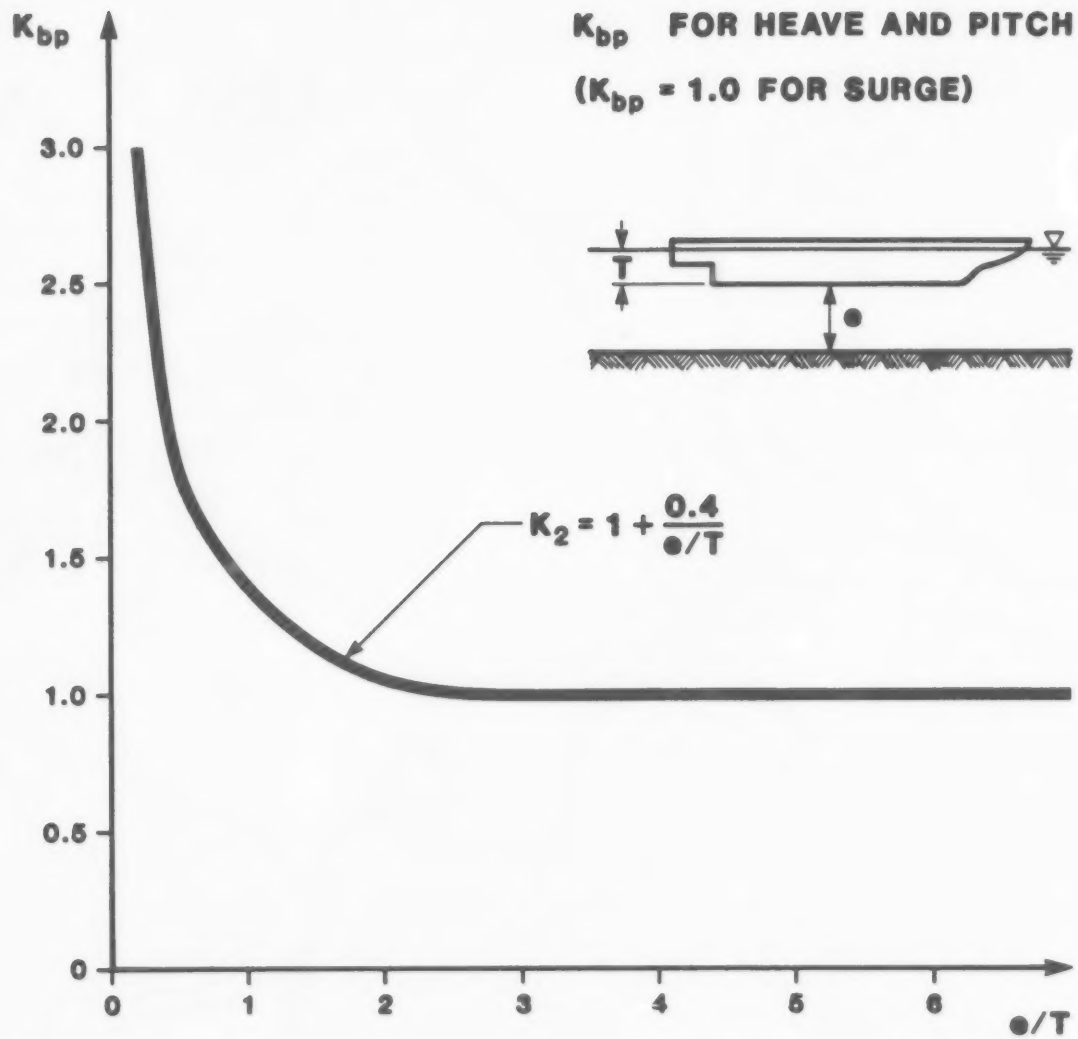


Fig. 29 Bottom Proximity Effect

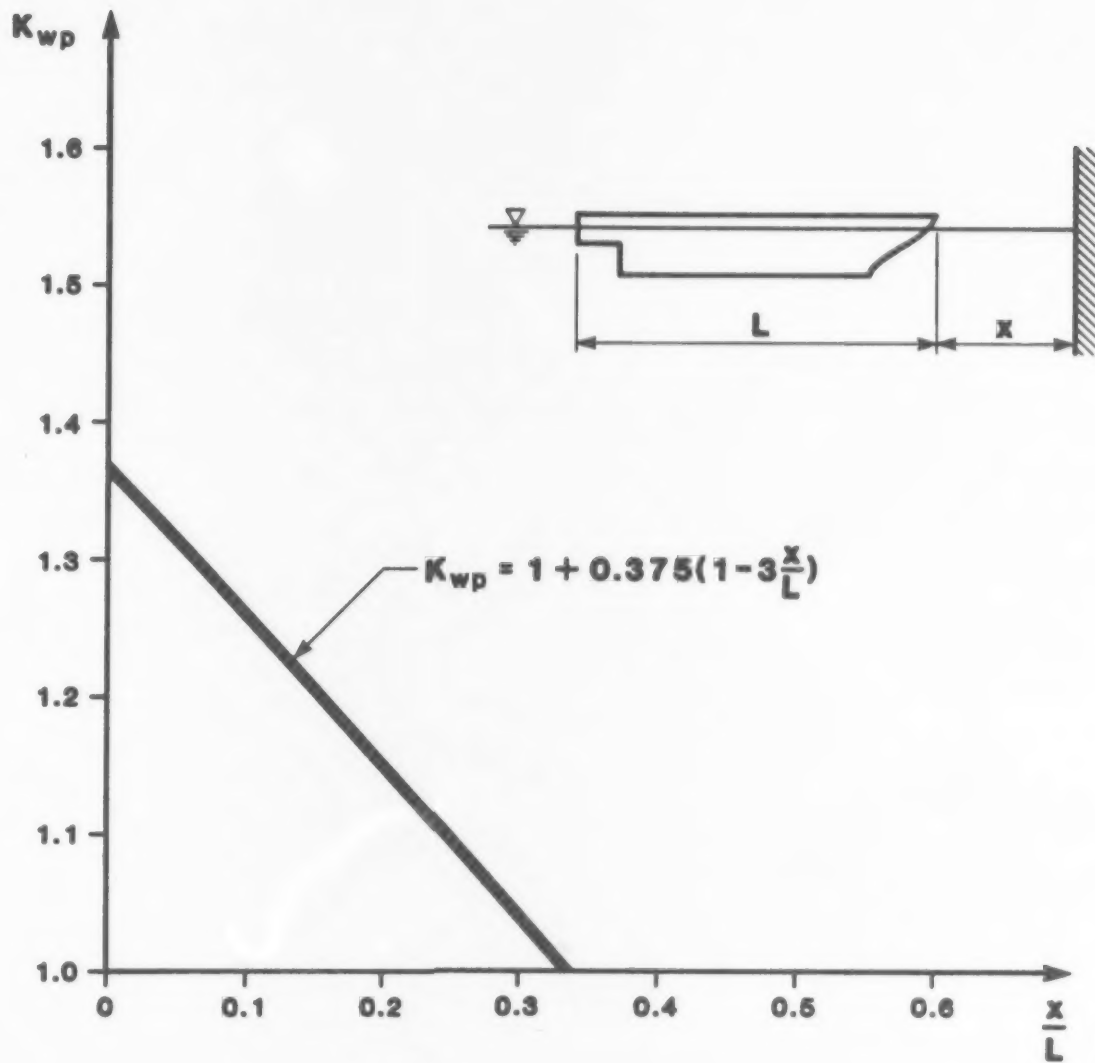


Fig. 30 Ice Proximity Effect

### 3.3 Full Scale data

The full scale data of the Kigoriak were recorded on a digital data acquisition system and the analysis was carried out by developing a software package written in HPL (the only language available in 1982 for the HP 9836 computer system). This program was revised to include the added mass algorithm presented in Section 2.3.

The results are most efficiently presented using the video display which can show the time variation of the calculated vertical added mass along the length of the ship. Only a sample of the available full scale data was analysed. Fourteen Kigoriak rams were reanalysed using the procedure described in Section 2.3 for establishing the time variation of added mass. Figures 31 and 32 show sample results of the vertical (combined heave and pitch) obtained for ram KR369. Figure 31a shows the time history of the measured vertical bowforce. This force is calculated from measured shear and normal strains at several sections along the hull and is estimated to have an error bound of about  $\pm 20$  per cent. This error is due to the assumptions involved in the integration of the measured strains to arrive at the cross sectional shear and bending actions and in the interpolation and extrapolation of the shear force values to the estimated location of the bow force.

The accelerations shown in Fig. 31b are those calculated at frames 27 and 12 using the measured bow and stern accelerations and the flexing accelerations according to the procedure described in Section 2.3.

The hull girder was divided into 10 sections and the vertical added mass associated with each section was calculated at every time instance. The values at time step No. 34 ( $t = 0.86$  s) are plotted in Fig. 31c.

It should be noted that the calculated vertical added masses were reasonable except at the moments in time and locations along the length of the ship where the vertical accelerations approaches the zero value. This is due to the fact that in Section 2.3, the force was divided by the acceleration to obtain the total mass which was then reduced by subtracting the actual mass to obtain the added mass component.

The acceleration traces clearly indicated that as soon as the impact phase is almost over, the accelerations tend to oscillate around the zero value giving rise to unreasonably high values for the added mass. Therefore the reader is cautioned against using these results directly in design. Further analysis of the data is required before reaching this stage.

Therefore, the present analysis of the data had to be carried out with great care in order to be able to disregard these numerically unstable regions. Theoretically, the force at the location or instant when the acceleration diminishes should also be zero. The error in the force calculations gives rise to nonzero values for the force which in turn give unrealistically high added mass values. In addition, the accelerations themselves do not purely measure the global response being influenced by local structural vibrations. A Fourier Transform analysis could be employed for identifying and filtering out these frequencies from the measurements. It was not however possible to carry out this work within the scope of the present study.

Figure 32 shows the variation of the added mass within three time steps with 0.04 s time step. The figure shows significant fluctuations within this short time period. The same observation was noted from the model test results (Section 3.4).

Figures 33 through 37 give further results obtained for ram KR369. These figures show the bending moment, shear force and distributed load distributions at time instances in the neighbourhood of the peak impact force (at time 0.64 s or time step No. 32 the peak force was 21.1 MN). The time step is 0.02 s corresponding to the 50 Hz data sampling rate. Inertia force and deflection time histories are shown in Fig. 34 together with the calculated virtual force. The virtual force is equivalent to the added mass times the acceleration. This quantity is shown, Fig. 37, to be more stable compared to the virtual added mass since the division by very small acceleration values is avoided. Figure 35 shows the hull deflection and acceleration profiles and Figure 36 shows the distributed added mass which also suffers from the instability inherent in the point added masses. Figure 37 shows snap shots of the virtual force at several consecutive time steps.

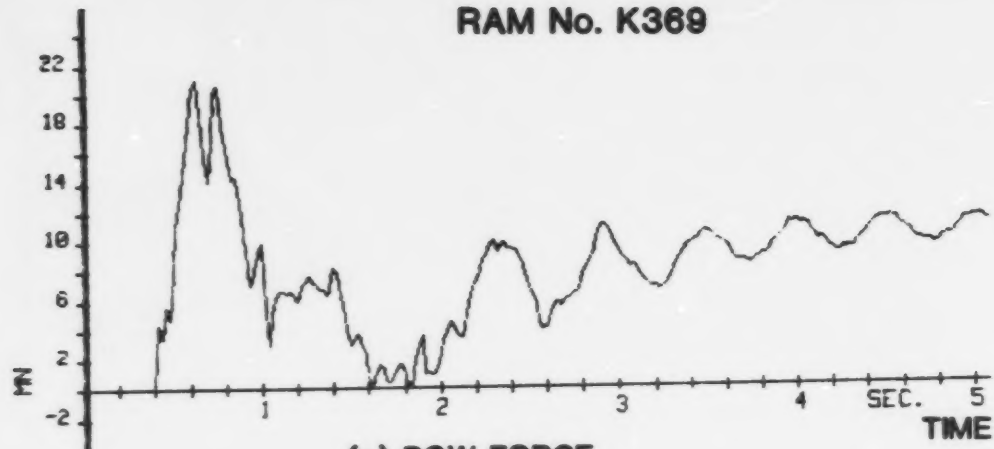
Figures 38 through 43 show results of added mass calculations for six further rams. Added mass coefficients were calculated for a total of 14 rams. In addition to the total vertical added mass coefficients, surge added mass coefficients were calculated using the methodology given in section 2.3 at the peak impact point. These results and further results from the full scale data analysis are given in Appendix C.

The following is a summary of the results arrived at from the full scale data analysis:

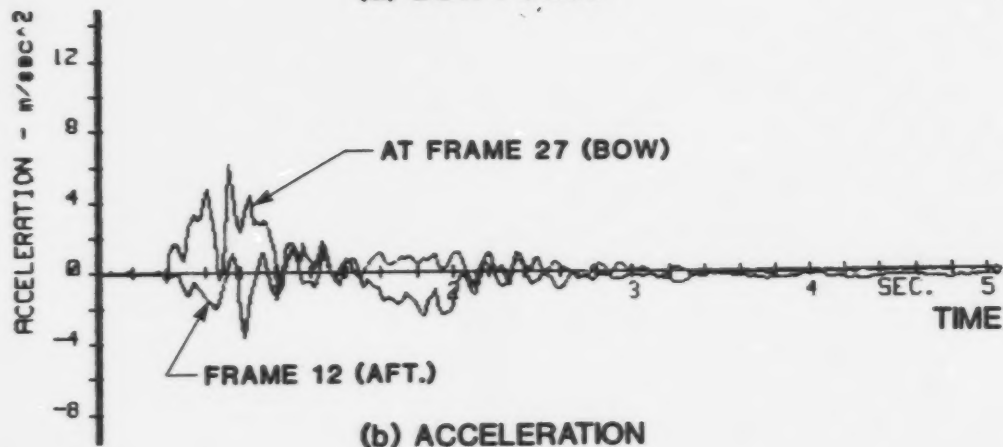
- 1) Erroneous added mass coefficients occur whenever very small accelerations do.
- 2) The variation of the vertical added mass along the hull is strongly time dependent.
- 3) The data in Figs. 38 - 43, excluding the unstable values, give vertical (combined heave, pitch, and flexural vertical modes of vibration) added mass coefficients at impact of 1.04 varying between 0.9 and 1.3.
- 4) The surge added mass coefficients vary between - 0.1 and 0.5 (based on an analysis of 16 rams). An average value of 0.2 is obtained.



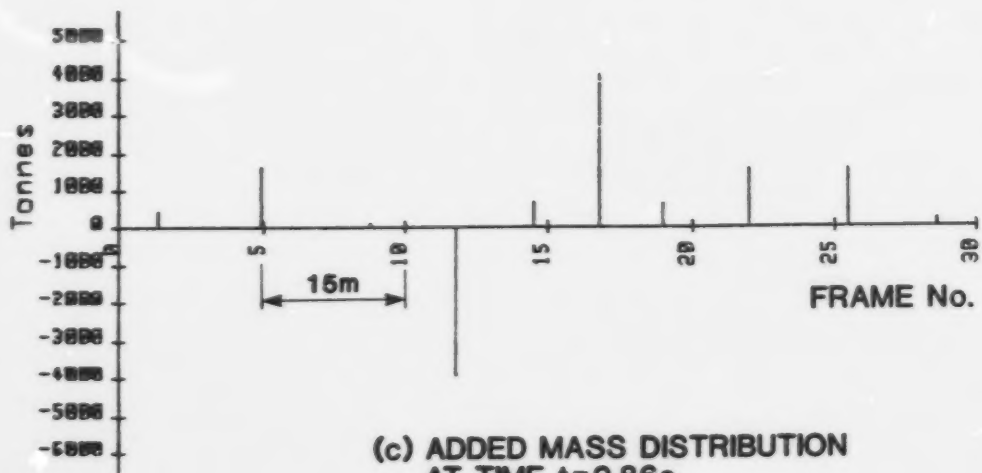
RAM No. K369



(a) BOW FORCE



(b) ACCELERATION



(c) ADDED MASS DISTRIBUTION  
AT TIME  $t=0.86s$

Fig. 31 Added Mass Variation Along the Hull (KR369)

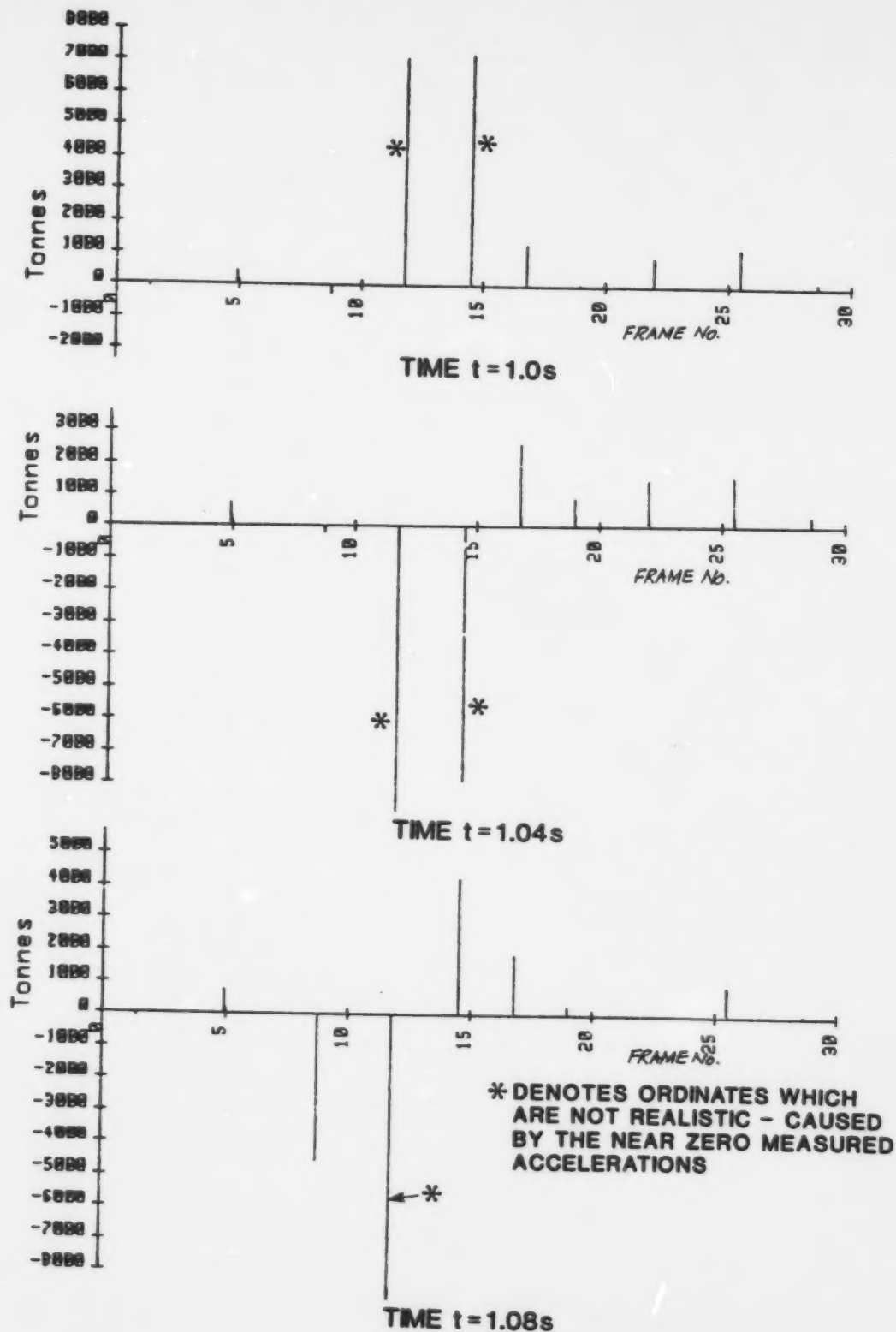
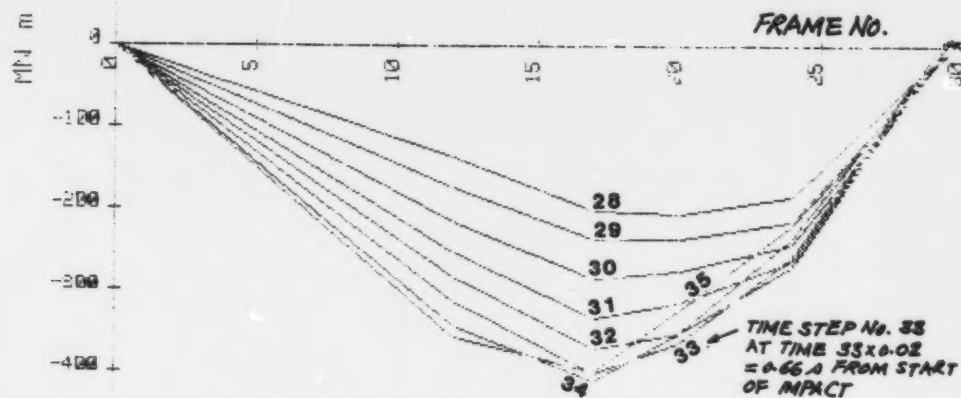
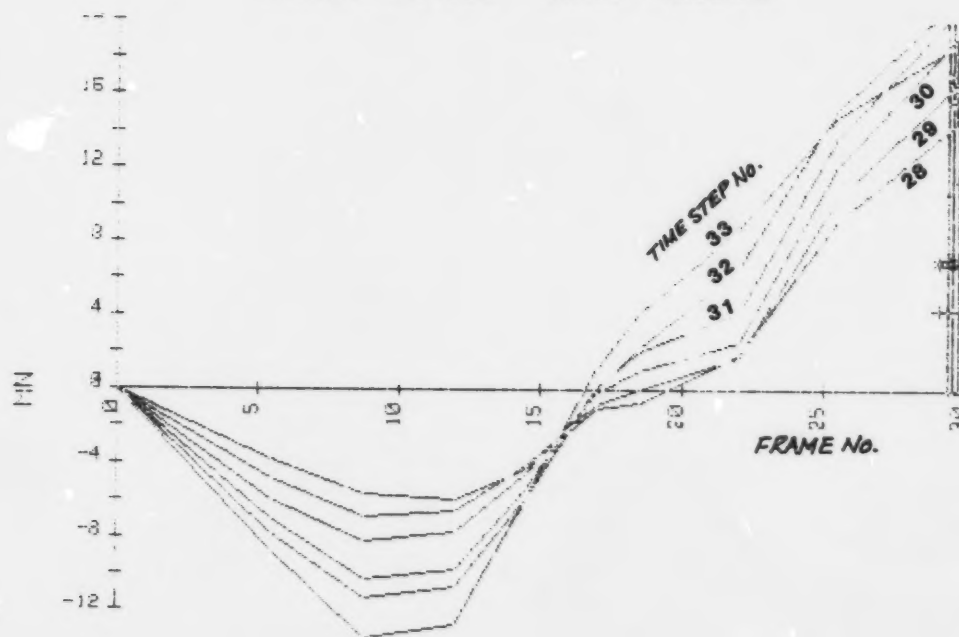


Fig. 32 Time Dependence of Vertical Added Mass (KR369)

# BENDING MOMENT - RAM #KR369



# SHEAR FORCE - RAM #KR369



# INERTIAL FORCE - RAM #KR369

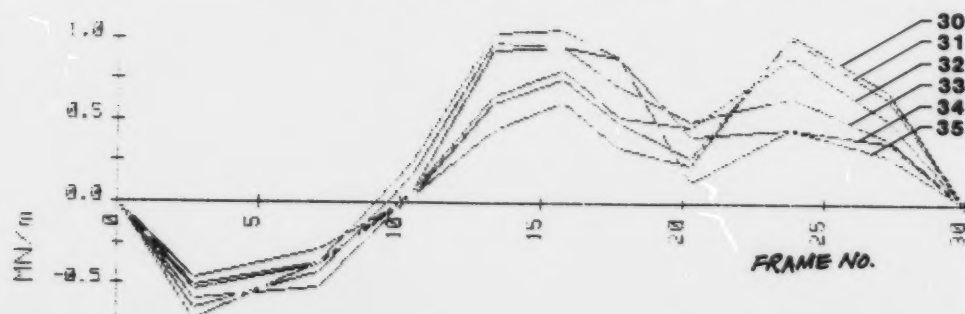


Fig. 33 Bending Moment Shear Force and Inertia Force (KR369)

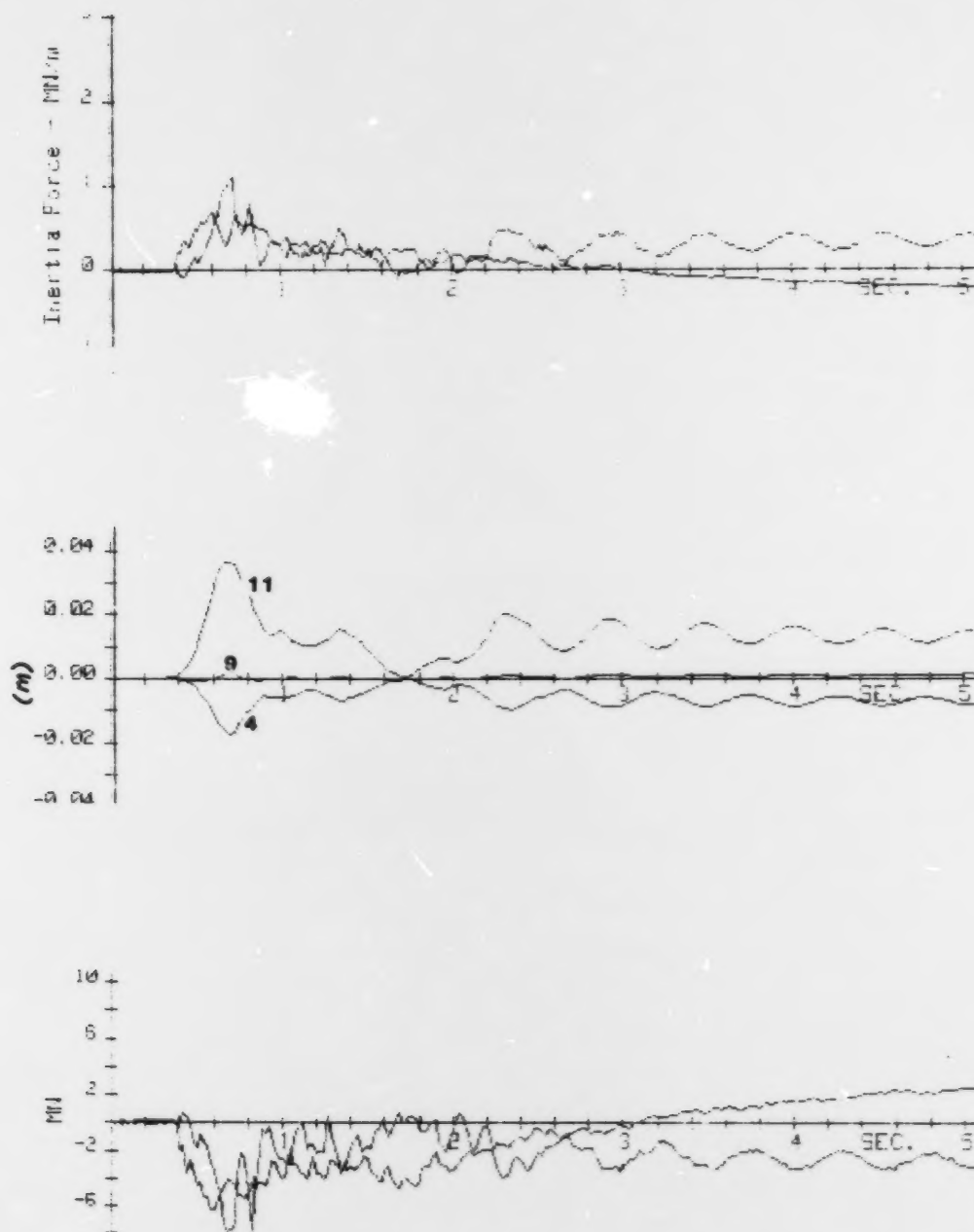
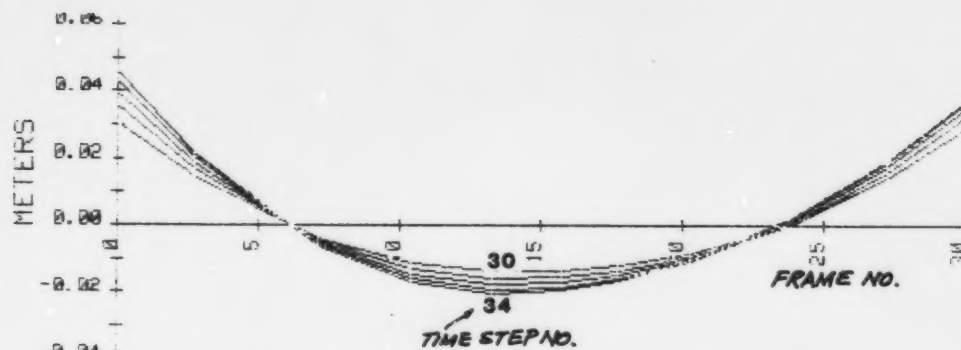
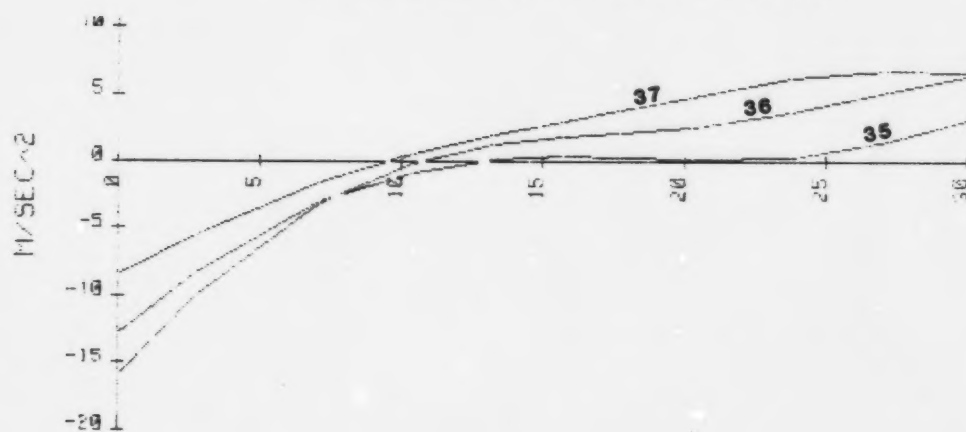


Fig. 34 Inertia Force, Deflection and Virtual Force Time Histories (KR369)

### DEFLECTION - RAM #KR369



### ACCELERATION - RAM #KR369



### ACCELERATION - RAM #KR369

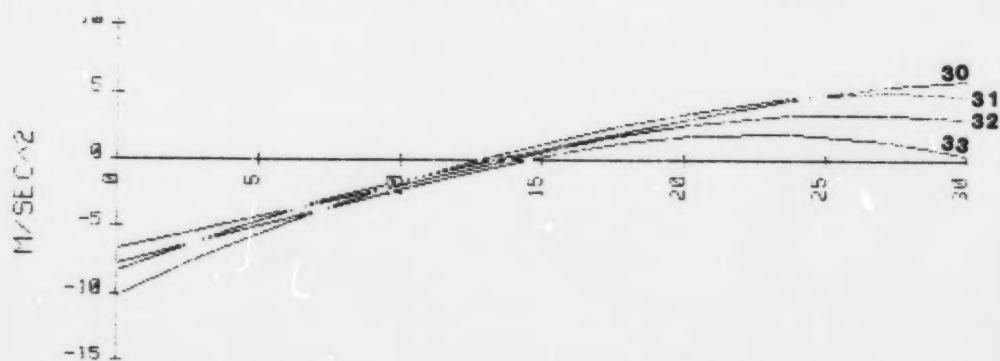


Fig. 35 Vertical Deflection and Acceleration (KR369)

**DIST ADDED MASS - RAM #KR369**

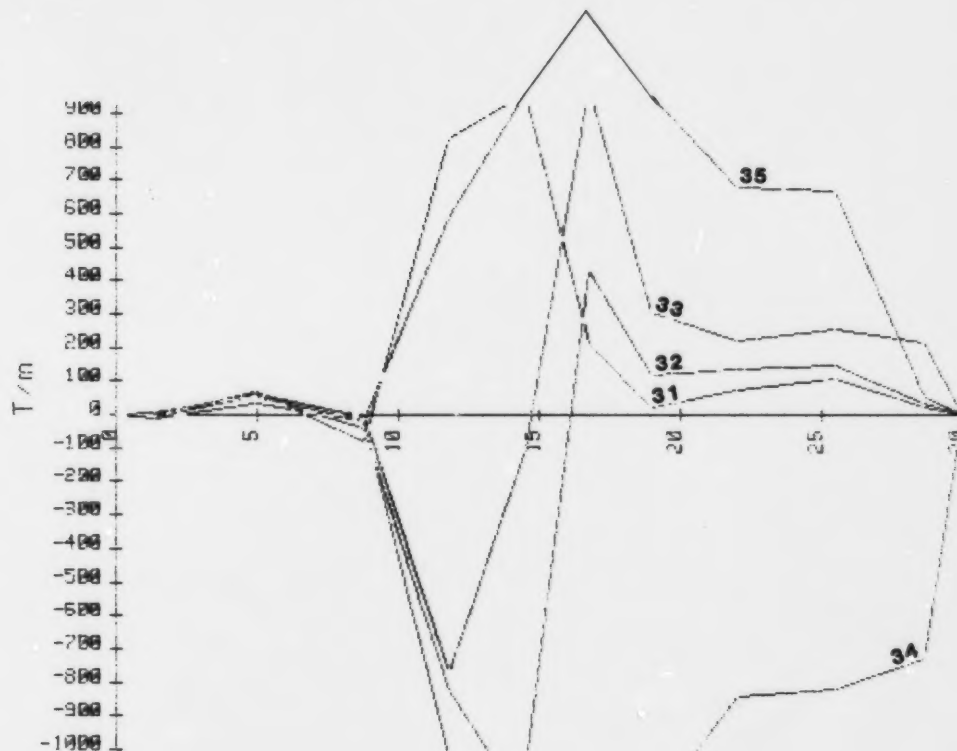
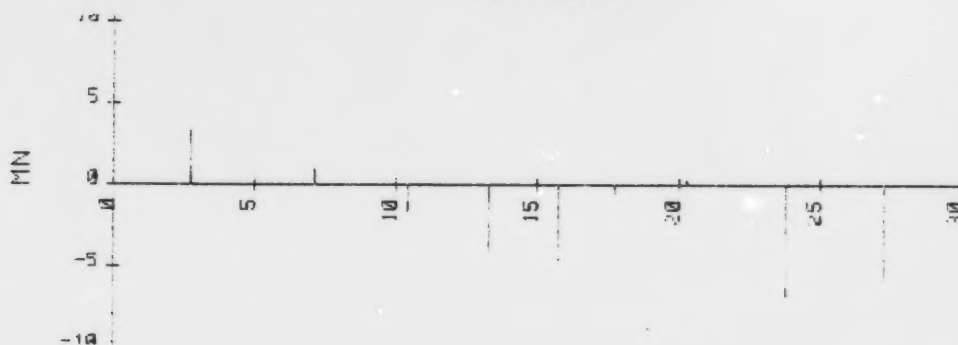


**Fig. 36 Distributed Added Mass (KR369)**

**VIRTUAL ADDED FORCE - RAM #KR369**  
**TIME STEP = 31**



**VIRTUAL ADDED FORCE - RAM #KR369**  
**TIME STEP = 32**



**VIRTUAL ADDED FORCE - RAM #KR369**  
**TIME STEP = 33**

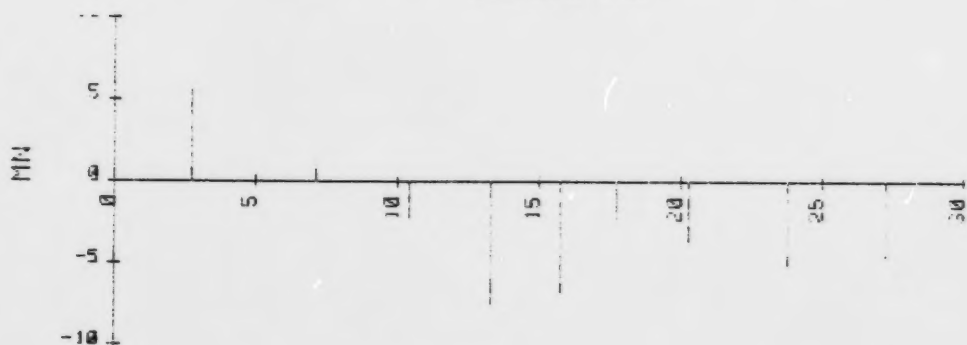


**Fig. 37 Virtual Added Force (KR369)**



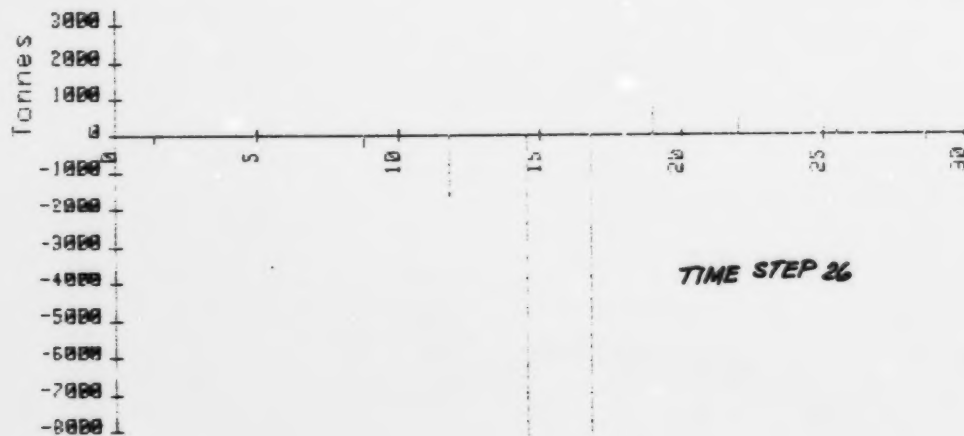
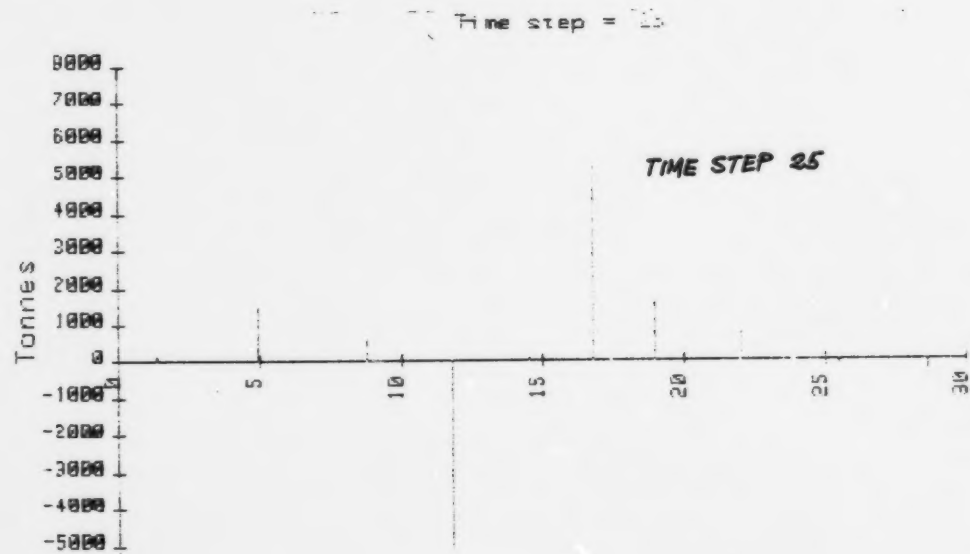
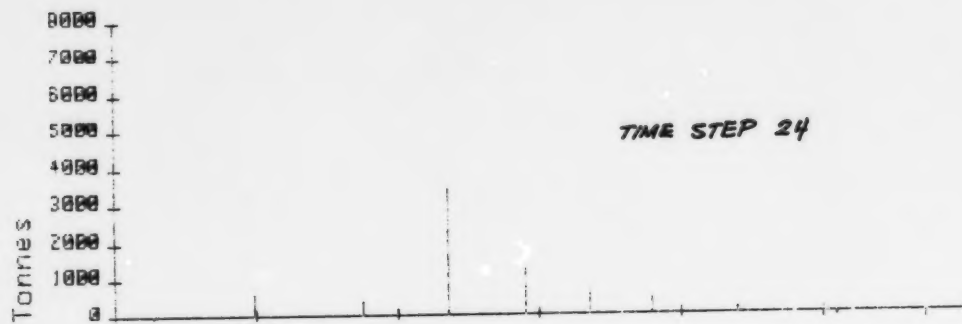


Fig. 38 Added Mass Distribution (Ram KR 355)

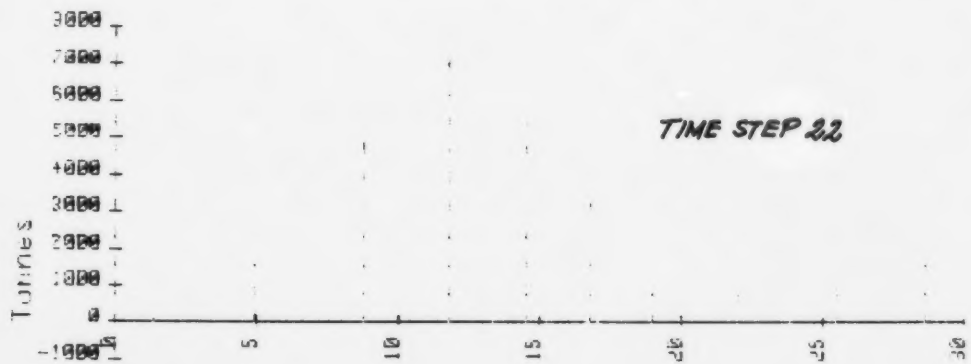
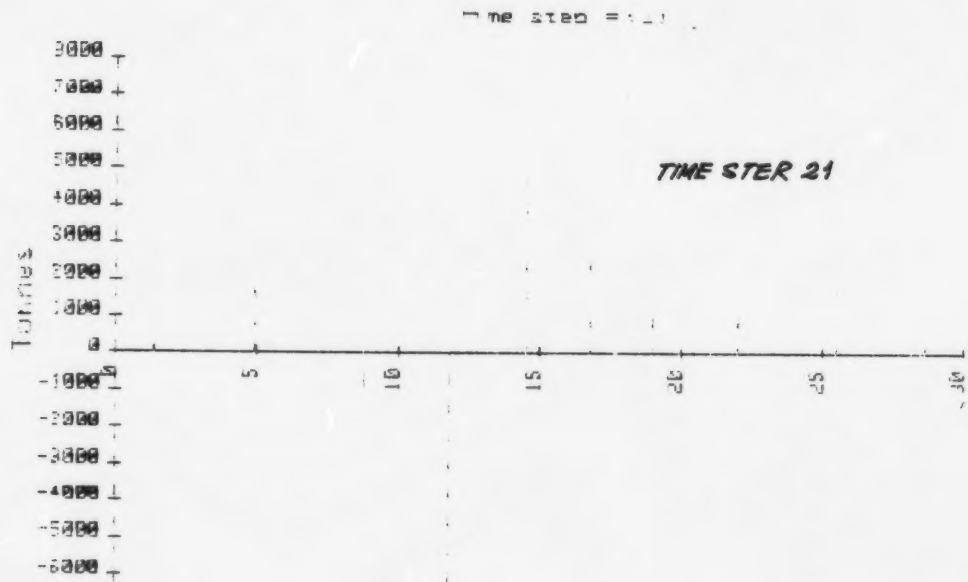
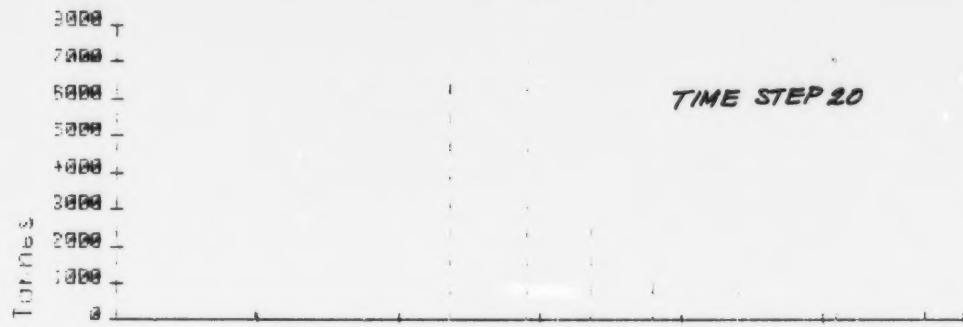
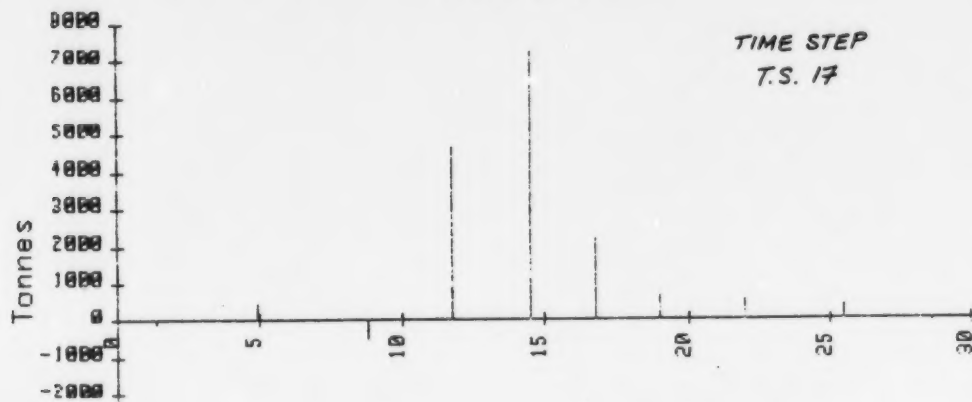
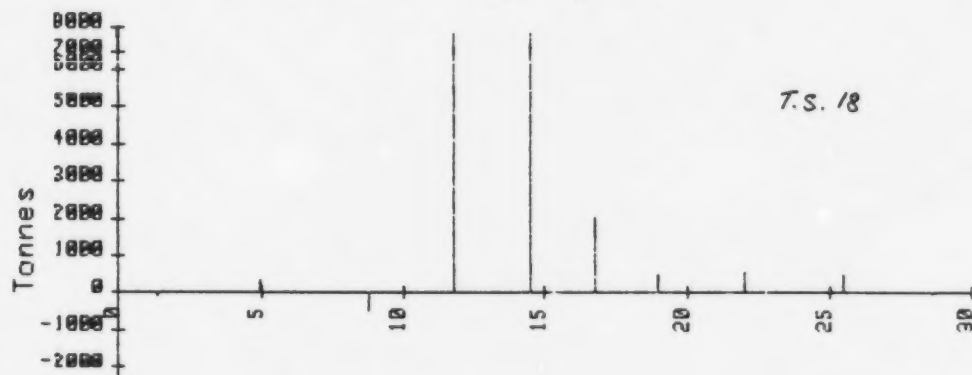


Fig. 39 Added Mass Distribution (Ram KR 370)

POINT ADDED MASS RAM #KR375  
Time step = 17



POINT ADDED MASS RAM #KR375  
Time step = 18



POINT ADDED MASS RAM #KR375  
Time step = 19

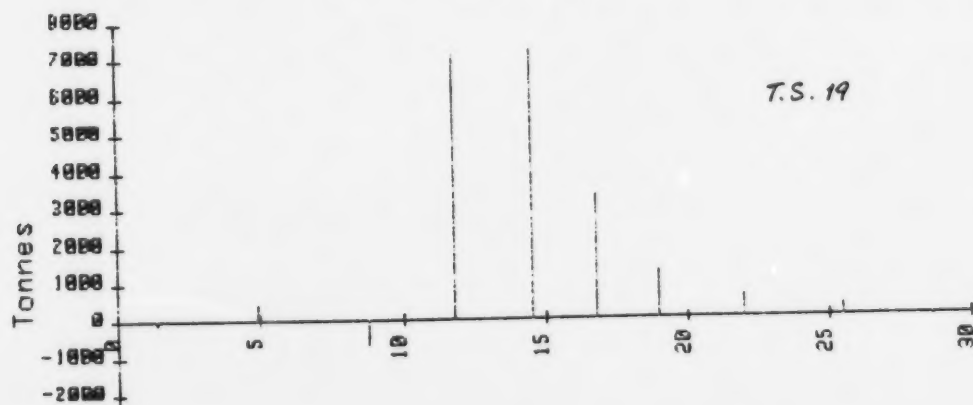
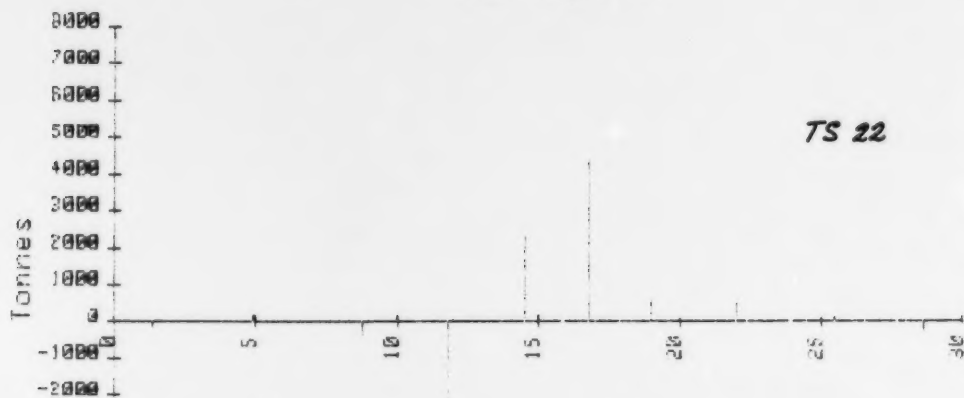
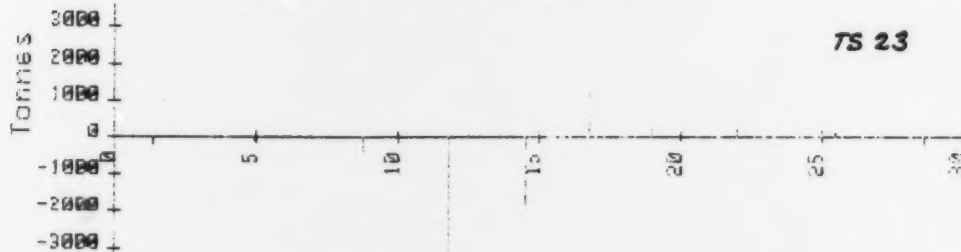


Fig. 40 Added Mass Distribution (Ram KR 375)

POINT ADDED MASS RAM #KR381  
Time step = 22



POINT ADDED MASS RAM #KR381  
Time step = 23



POINT ADDED MASS RAM #KR381  
Time step = 24

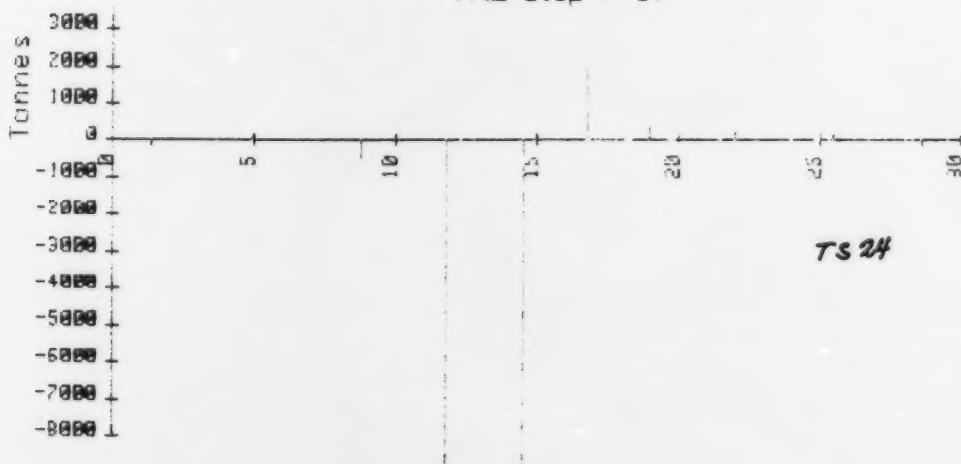


Fig. 41 Added Mass Distribution (Ram KR 381)

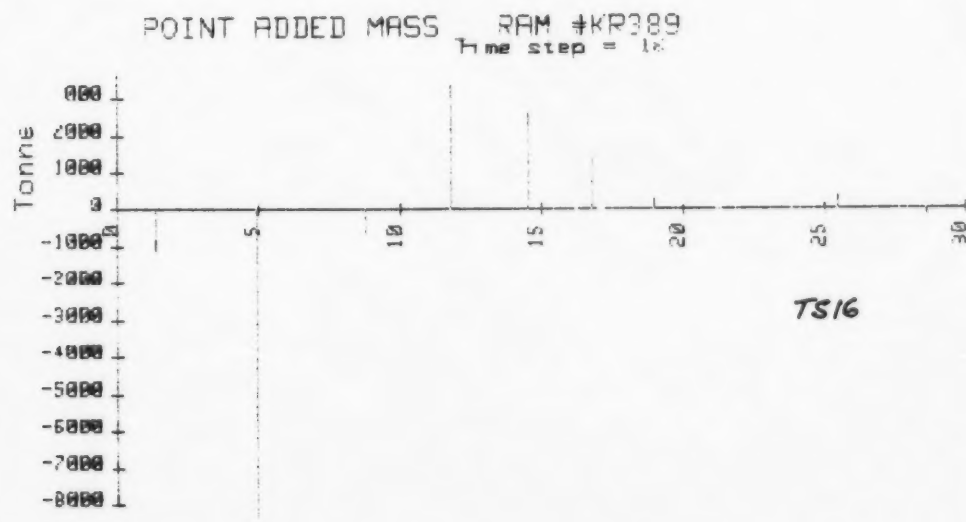
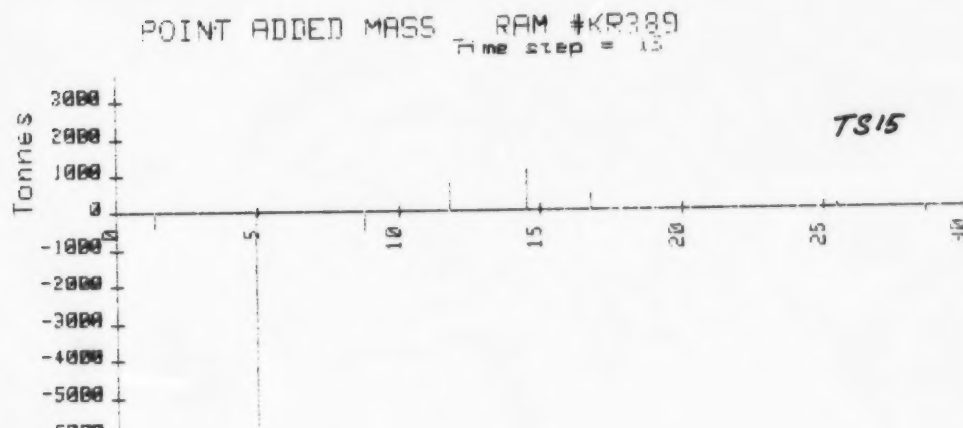
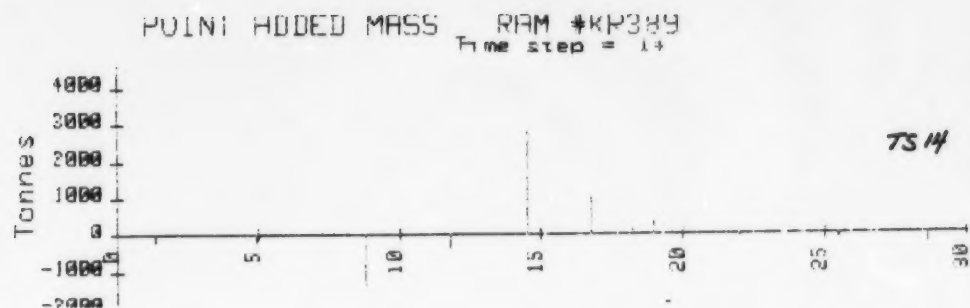


Fig. 42 Added Mass Distribution (Ram KR 389)

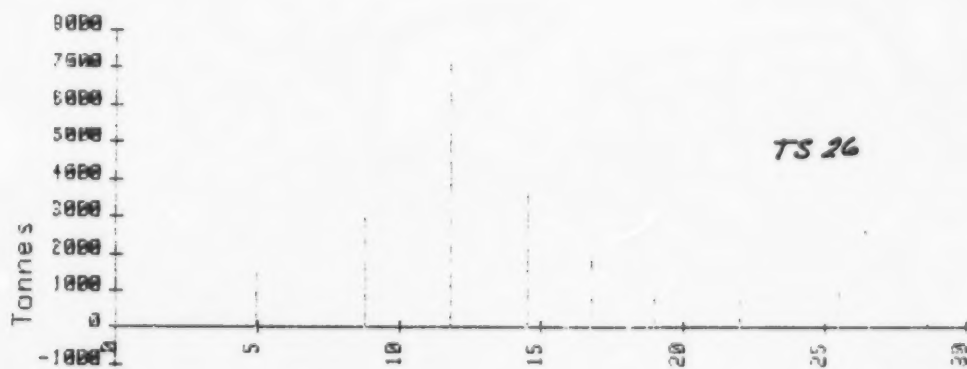
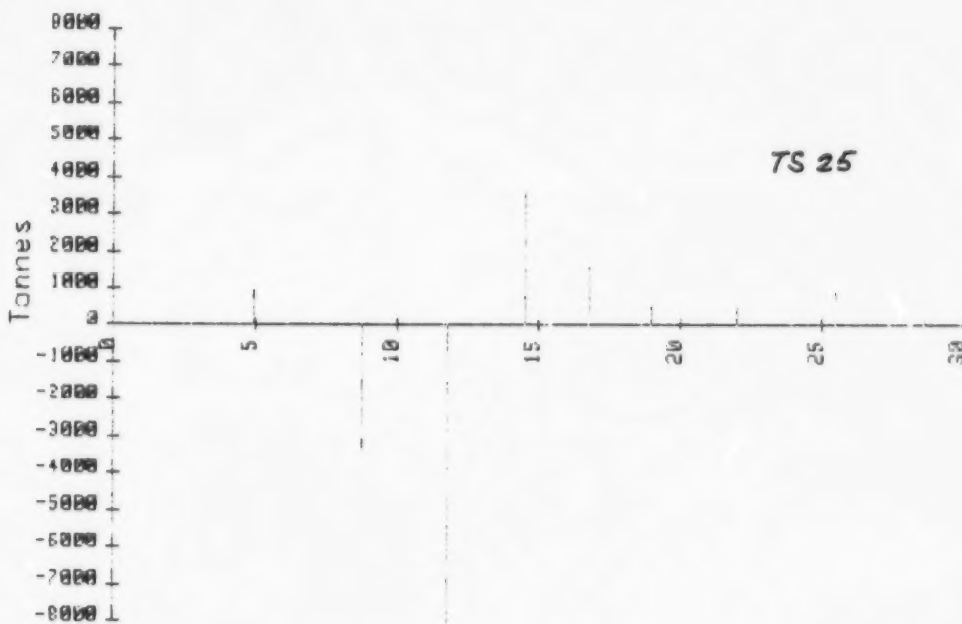
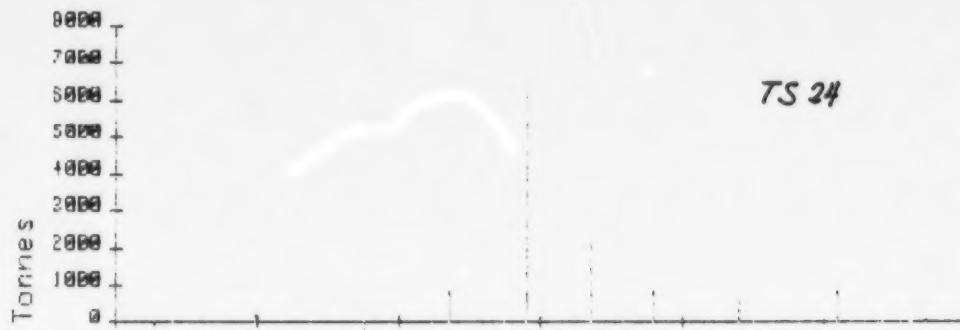


Fig. 43 Added Mass Distribution (Ram KR 400)

### 3.4 Model Tests

This section presents the results obtained from representative model tests of the M.V. Arctic and the M.V. Kigoriak. The testing program was explained in Section 2.4 and the full details of the test arrangement, parameters, data and photographs are given in Appendix B.

Table 5 gives an overview of the tests carried out on both the M.V. Arctic and the Canmar Kigoriak models. Tables 6 and 7 give detailed description of the parameters varied in each test as well as the value of the measured impulse or impact forces for the M.V. Arctic and Kigoriak models respectively.

**TABLE 5**  
**Model Tests**

| Vessel        | Test No. | Type of Test    | No. of Runs |
|---------------|----------|-----------------|-------------|
| M.V. Arctic   | 3845     | Impulse/No Floe | 22          |
|               | 3847     | Impulse/No Floe | 32          |
|               | 3848     | Impulse/Floe    | 24          |
|               | 3855     | Impact/3.5 MPa  | 27          |
|               | 3856     | Impact/14 MPa   | 28          |
|               | 3857     | Impact/35 MPa   | 13          |
| M.V. Kigoriak | 3858     | Impact/4.7 MPa  | 30          |
|               | 3859     | Impact/9.3 MPa  | 31          |
|               | 3860     | Impact/15.5 MPa | 12          |
|               | 3861     | Impulse/No Floe | 22          |
|               | 3862     | Impulse/Floe    | 24          |



TABLE 6  
M.V. Arctic Model Tests

| TEST NO. 3845 |       |                        |            |        |        |            |  |  |       |
|---------------|-------|------------------------|------------|--------|--------|------------|--|--|-------|
| RUN NO.       | TYPE  | MONITORING<br>TIME (s) | RATE (F/S) | Bp (m) | V (kn) | Fmax (kgf) |  |  |       |
| 1             | TRIAL | 5.48                   |            |        |        |            |  |  |       |
| 2             |       | 37.15                  |            |        |        |            |  |  |       |
| 3             |       | 3.91                   |            |        |        |            |  |  |       |
| 4             | SURGE | 35.29                  | S (SLOW)   |        | 12     | 0          |  |  |       |
| 5             | PITCH | 12.89                  | F (FAST)   |        | 12     |            |  |  |       |
| 6             | HEAVE | 12.38                  |            |        | 12     |            |  |  |       |
| 7             | PITCH | 9.52                   | F          |        | 12     | 0          |  |  |       |
| 8             | SURGE | 13.78                  | F          |        | 12     | 0          |  |  |       |
| 9             | PITCH | 8.77                   | S          |        |        |            |  |  | 55    |
| 10            | PITCH | 11.74                  | F          |        | 12     | 0          |  |  | 60.7  |
| 11            |       | 8.68                   |            |        | 12     |            |  |  | 41.3  |
| 12            |       | 8.65                   |            |        | 12     |            |  |  | 116.8 |
| 13            | SURGE | 39.05                  |            |        | 30     | 0          |  |  | 148   |
| 14            | HEAVE | 10.55                  |            |        | 30     | 0          |  |  |       |
| 15            | PITCH | 6.03                   | F          |        | 30     | 0          |  |  |       |
| 16            | SURGE | 25.58                  |            |        | 30     |            |  |  | 115   |
| 17            |       | 16.05                  |            |        | 30     | 0          |  |  | 44.2  |
| 18            | PITCH | 22.14                  | F          |        | 30     |            |  |  | 36.6  |
| 19            | SURGE | 12.92                  | F          |        | 50     |            |  |  |       |
| 20            | HEAVE | 6.9                    |            |        | 50     |            |  |  |       |
| 21            | PITCH | 7.18                   |            |        | 50     | 0          |  |  |       |
| 22            |       | 7.65                   |            |        | 50     | 0          |  |  |       |

Complete Table at the end of the Section (Pages 143 - 148)



Initially the plan was to analyse the data in the time domain to arrive at the added mass at each instant of time throughout each impact scenario. However, it became evident during this exercise, Table 8, that such an approach requires computer and manpower resources which are significantly larger than those available for this project. Table 8 gives an example of the variation of the heave added mass coefficient with time for Run No. 2 of Test No. 3862 (Impulse Floe). These results were obtained using the MULTICS computer system of the University of Calgary. The added mass coefficients shown are unrealistically high or low due to the same reasons given previously with regard to the full scale test data analysis. That is; the acceleration were small at the centre of gravity compared to the bow and stern accelerations and in many instances is very close to zero. Combined with the inaccuracies of the measurements and the estimation of the forces, these lead to the shown variability in added mass. A simpler yet efficient approach was subsequently adopted which utilized the capabilities available within the personal micro computer industry. This was achieved by employing an IBM Personal Computer and the Lotus 1.2.3 software package.

Tests No. 3856 (Impact/14 MPa) and 3848 (Impulse/Floe) for the M.V. Arctic were analysed in detail, as samples of ramming and forced motions tests, shown in Tables 9 and 10. This analysis was carried out using the Lotus 1.2.3 integrated software and using an IBM Personal Computer. Similarly tests No. 3859 (Impact/9.3 MPa) and 3862 (Impulse/Floe) were analysed for the M.V. Kigoriak and the results are shown in Tables 11 and 12.

Table 9 shows the run number, duration of the run, the ice strength and thickness followed by the bottom proximity Bp and the impact velocity. A blank in this range of columns indicates a value equivalent to the last value directly above in the column. The table then shows two sets of results eight columns each. The first eight columns show data representing the impact phase whereas the next eight columns give results for the beaching phase. The beaching phase is defined as the phase following the initial impact when the ship is sliding up on the ice. The ratio between the horizontal and vertical impact forces are given and a comment column is shown at the right most hand side of the table. The symbols D1, D2 and D3 refer to surge, bow heave and stern accelerations, respectively.

TABLE 8  
Heave Added Mass Coefficients (Test 3862, Run 2)

| Count<br>I | Time<br>(s) | Total<br>Force<br>$F_z$<br>(N) | Hydrostatic<br>Force<br>$K_z$<br>(N) | C.G.<br>Acceleration<br>$Z$<br>(m/s <sup>2</sup> ) | Added Mass<br>Coefficient<br>$\lambda_z$ |
|------------|-------------|--------------------------------|--------------------------------------|----------------------------------------------------|------------------------------------------|
| 106        | 2.12        | -4.9127                        | -25.7649                             | -0.0636                                            | -0.4505                                  |
| 107        | 2.14        | -4.9127                        | -9.1617                              | -0.0512                                            | -0.8611                                  |
| 108        | 2.16        | 9.5649                         | 4.7135                               | -0.0042                                            | 0.9384                                   |
| 109        | 2.18        | -4.9127                        | -19.5679                             | -0.0007                                            | 36.0967                                  |
| 110        | 2.20        | -4.9127                        | -7.1799                              | -0.0281                                            | -0.8650                                  |
| 111        | 2.22        | 110.9002                       | -12.3842                             | -1.0350                                            | -0.8005                                  |
| 112        | 2.24        | 530.7682                       | -5.1969                              | -0.4172                                            | 1.1518                                   |
| 113        | 2.26        | 907.1779                       | -3.7108                              | -0.1266                                            | 11.0542                                  |
| 114        | 2.28        | 1037.9452                      | -7.4278                              | -0.9905                                            | 0.7678                                   |
| 115        | 2.30        | 747.9616                       | 0.2535                               | -0.2436                                            | 4.1407                                   |
| 116        | 2.32        | 328.0936                       | 17.0967                              | -0.1302                                            | 3.0000                                   |
| 117        | 2.34        | -4.9127                        | 43.1120                              | -0.4218                                            | -1.1907                                  |
| 118        | 2.36        | -178.6910                      | 53.7733                              | 0.2987                                             | 0.3037                                   |
| 119        | 2.38        | -236.5700                      | 44.3514                              | 0.0715                                             | 5.5800                                   |
| 120        | 2.40        | -236.5700                      | 71.8589                              | 0.1738                                             | 1.9729                                   |
| 121        | 2.42        | -178.6910                      | 90.1972                              | 0.0686                                             | 5.5665                                   |
| 122        | 2.44        | -62.0251                       | 97.1282                              | 0.1857                                             | 0.4426                                   |
| 123        | 2.46        | 53.0016                        | 102.8318                             | -0.2481                                            | -1.3365                                  |
| 124        | 2.48        | 168.7792                       | 119.4253                             | 0.0960                                             | -1.0608                                  |
| 125        | 2.50        | 313.5748                       | 122.7103                             | -0.2340                                            | 0.3664                                   |
| 126        | 2.52        | 414.9121                       | 132.0961                             | -0.0624                                            | 6.5907                                   |
| 127        | 2.54        | 429.4309                       | 157.3654                             | -0.0943                                            | 3.8334                                   |
| 128        | 2.56        | 458.3704                       | 167.2324                             | -0.0986                                            | 3.9458                                   |
| 129        | 2.58        | 458.3704                       | 178.6638                             | -0.0370                                            | 11.6487                                  |
| 130        | 2.60        | 458.3704                       | 185.2819                             | -0.1192                                            | 2.8366                                   |
| 131        | 2.62        | 443.9497                       | 189.4935                             | -0.1258                                            | 2.3874                                   |
| 132        | 2.64        | 443.9497                       | 199.7215                             | -0.0402                                            | 9.1885                                   |
| 133        | 2.66        | 429.4309                       | 227.7584                             | 0.0351                                             | -10.6154                                 |
| 134        | 2.68        | 429.4309                       | 242.5590                             | 0.0371                                             | -9.4460                                  |
| 135        | 2.70        | 443.9497                       | 246.5299                             | 0.0636                                             | -6.1994                                  |
| 136        | 2.72        | 458.3704                       | 278.7783                             | -0.0791                                            | 2.8014                                   |
| 137        | 2.74        | 487.3099                       | 284.9152                             | -0.0062                                            | 53.6604                                  |
| 138        | 2.76        | 516.2494                       | 337.4994                             | 0.0187                                             | -16.9933                                 |
| 139        | 2.78        | 530.7682                       | 312.7114                             | 0.0547                                             | -7.6820                                  |
| 140        | 2.80        | 545.2870                       | 354.5862                             | 0.0547                                             | -6.8437                                  |
| 141        | 2.82        | 545.2870                       | 321.1345                             | -0.0196                                            | 18.1250                                  |
| 142        | 2.84        | 545.2870                       | 375.1627                             | 0.0380                                             | -8.4898                                  |
| 143        | 2.86        | 545.2870                       | 346.6445                             | 0.0556                                             | -6.9887                                  |
| 144        | 2.88        | 559.7077                       | 356.2709                             | 0.0826                                             | -5.1276                                  |
| 145        | 2.90        | 545.2870                       | 349.1714                             | 0.0553                                             | -6.9380                                  |

TABLE 9

M.V. Arctic I.2.3 Worksheet for Test No. 3856 (Impact/14 MPa)  
(Impact Phase)

TEST NO. 3856

| RUN NO. | TIME(s) | SIG(MPa) | Hice(m) | Bp(m) | V (Kn) | Fx(kgf) | Fz(kgf) | D11    | D2I    | D3I    | AMXI     | AMZI     |
|---------|---------|----------|---------|-------|--------|---------|---------|--------|--------|--------|----------|----------|
| 1       | 10.52   | 14       | 15      | 15    | 50     | 22.8    | 32.9    | 0.15   | 0.3    | -0.29  | -0.49668 | 20.78807 |
| 2       | 8.63    | 14       | 15      | 15    | 50     | 69.7    | 86.3    | 0.4    | 1      | -0.64  | -0.42301 | 0.587564 |
| 3       | 8.2     | 14       | 15      | 15    | 50     | 110     | 128.9   | 0.73   | 1.1    | -1.125 | -0.50104 | -35.1456 |
| 4       | 10      | 14       | 15      | 15    | 30     | 21.9    | 26.9    | 0.145  | 0.27   | -0.22  | -0.49988 | 2.562913 |
| 5       | 7.86    | 14       | 15      | 15    | 30     | 64.9    | 83.75   | 0.375  | 0.8    | -0.74  | -0.42693 | 8.243929 |
| 6       | 7.76    | 14       | 15      | 15    | 30     | 82      | 112.5   | 0.33   | 1.25   | -1.06  | -0.17720 | 2.921226 |
| 7       | 11.46   | 14       | 15      | 15    | 12     | 20.6    | 25.8    | 0.137  | 0.28   | -0.201 | -0.50210 | 1.162796 |
| 8       | 9.92    | 14       | 15      | 15    | 12     | 53.6    | 77.45   | 0.32   | 0.66   | -0.54  | -0.44536 | 3.274282 |
| 9       | 7.24    | 14       | 15      | 15    | 15     | 100     | 111     | 0.665  | 0.9625 | -0.95  | -0.50206 | 57.80794 |
| 10      | 10.22   | 14       | 10      | 10    | 12     | 21.9    | 27.2    | 0.13   | 0.222  | -0.15  | -0.44218 | 1.501839 |
| 11      | 8.4     | 14       | 10      | 10    | 12     | 64.4    | 77.4    | 0.344  | 0.64   | -0.66  | -0.38010 | -26.6291 |
| 12      | 7.4     | 14       | 10      | 10    | 12     | 107.5   | 147.5   | 0.6    | 0.9125 | -1     | -0.40673 | -12.1636 |
| 13      | 9.52    | 14       | 10      | 10    | 30     | 19.3    | 31      | 0.12   | 0.24   | -0.21  | -0.46743 | 5.843267 |
| 14      | 7.8     | 14       | 10      | 10    | 30     | 71.1    | 87.2    | 0.3    | 0.64   | -0.66  | -0.21523 | -29.8741 |
| 15      | 6.84    | 14       | 10      | 10    | 30     | 105     | 140     | 1.16   | 0.92   | -1.16  | -0.70027 | -4.86313 |
| 16      | 10.04   | 14       | 10      | 10    | 50     | 17.2    | 27.2    | 0.125  | 0.25   | -0.206 | -0.54437 | 3.093919 |
| 17      | 7.66    | 14       | 10      | 10    | 50     | 57.1    | 79.2    | 0.28   | 0.56   | -0.68  | -0.32473 | -5.37086 |
| 18      | 7.16    | 14       | 10      | 10    | 50     | 96.8    | 39      | 0.8    | 0.84   | -0.76  | -0.59933 | 2.228476 |
| 19      | 7.66    | 14       | 10      | 10    | 50     | 105     | 120     | 0.844  | 0.8    | -0.76  | -0.58805 | 18.86754 |
| 20      | 10.8    | 14       | 5       | 5     | 50     | 15.25   | 19      | 0.0975 | 0.22   | -0.185 | -0.48208 | 2.595080 |
| 21      | 7.56    | 14       | 5       | 5     | 50     | 58.2    | 74.2    | 0.225  | 0.51   | -0.5   | -0.14348 | 48.13907 |
| 22      | 7.68    | 14       | 5       | 5     | 50     | 109     | 141.9   | 0.67   | 1      | -0.875 | -0.46130 | 6.517880 |
| 23      | 10.84   | 14       | 5       | 5     | 30     | 19.27   | 29.8    | 0.107  | -0.24  | -0.34  | -0.40366 | -1.34026 |
| 24      | 5.54    | 14       | 5       | 5     | 30     | 62.1    | 91.9    | 0.2325 | 0.56   | -0.55  | -0.11557 | 59.86092 |
| 25      | 6.74    | 14       | 5       | 5     | 30     | 100     | 140     | 0.52   | 1.14   | -0.98  | -0.36321 | 4.794701 |
| 26      | 9.58    | 14       | 5       | 5     | 12     | 15.12   | 26.1    | 0.0813 | 0.2375 | -0.2   | -0.38417 | 3.609271 |
| 27      | 7.58    | 14       | 5       | 5     | 12     | 64.8    | 90.1    | 0.225  | 0.64   | -0.56  | -0.04635 | 6.458609 |
| 28      | 6.52    | 14       | 5       | 5     | 15     | 120     | 140     | 0.415  | 1.3125 | -1.075 | -0.04252 | 2.903799 |



TABLE 9 (Continued)  
(Beaching Phase)

TEST NO. 3856

| RUN NO. | AMPI     | Fxb(kgf) | Fzb(kgf) | D1B    | D2B     | D3B    | AMXB     | AMZB     | AMPB     | Fx/Fz    |
|---------|----------|----------|----------|--------|---------|--------|----------|----------|----------|----------|
| 1       | 0.550203 | 20       | 12       | 0.06   | -0.01   | 0.08   | 0.103752 | 0.135288 | -4.70666 | 0.693009 |
| 2       | 0.462890 | 40       | 30       | 0.075  | -0.1    | 0.08   | 0.766004 | -10.9337 | -5.63333 | 0.807647 |
| 3       | 0.610525 | 179.8    | 127.5    | 1.275  | -1.5    | 0.875  | -0.53304 | -2.35099 | -2.49242 | 0.853374 |
| 4       | 0.526163 | 18.1     | 11       | 0.06   | -0.03   | 0.029  | -0.00110 | -73.8476 | -6.18305 | 0.814126 |
| 5       | 0.511850 | 29.2     | 25       |        |         |        |          |          |          | 0.774925 |
| 6       | 0.353896 | 162      | 137.6    | 0.95   | -0.6    | 0.45   | -0.43534 | -7.07505 | -4.64312 | 0.728888 |
| 7       | 0.491143 | 19.75    | 13.5     | 0.1    | 0       | 0      | -0.34602 |          |          | 0.798449 |
| 8       | 0.794258 | 47.4     | 19       | 0.36   | -0.36   | 0.33   | -0.56401 | -5.19426 | -1.76550 | 0.692059 |
| 9       | 0.613490 | 162.2    | 140.8    | 0.91   | -0.5625 | 0.36   | -0.40979 | -5.60469 | -5.24307 | 0.900900 |
| 10      | 1.032688 | 18.75    | 9        | 0.1    | -0.13   | 0.08   | -0.37913 | -2.19205 | -2.19142 | 0.805147 |
| 11      | 0.655169 | 39.2     | 19       | 0.3    | -0.44   | 0.36   | -0.56732 | -2.57284 | -1.66025 | 0.832041 |
| 12      | 1.144052 | 165.9    | 165.5    | 1.57   | -0.55   | 0.5    | -0.65010 | -22.9205 | -5.38180 | 0.728813 |
| 13      | 0.915111 | 18.75    | 14.6     | 0.1    | -0.15   | 0.1375 | -0.37913 | -8.73509 | -2.41175 | 0.622580 |
| 14      | 0.864738 | 31.2     | 20       | 0.2    | -0.46   | 0.784  | -0.48344 | -0.59120 | -1.44694 | 0.815366 |
| 15      | 0.871153 | 164.3    | 149.2    | 0.9    | 1.55    | -0.56  | -0.39551 | -0.00193 | 0.965763 | 0.75     |
| 16      | 0.658245 | 18.8     | 11.25    | 0.11   | -0.125  | 0.15   | -0.43407 | 1.980132 | -2.13727 | 1.671111 |
| 17      | 0.775612 | 30       | 17.5     | 0.234  | -0.39   | 0.4    | -0.57547 | 10.58940 | -1.61582 | 0.720959 |
| 18      | -0.32237 | 48.4     | 19.5     | 0.4    | -0.65   | 0.56   | -0.59933 | -2.43487 | -1.44801 | 2.482051 |
| 19      | 1.138461 | 148.5    | 127.4    | 1.0625 | 0.5     | -0.76  | -0.53720 | -4.24503 | 1.810888 | 0.875    |
| 20      | 0.304197 | 21.1     | 9.7      | 0.075  | -0.034  | 0.025  | -0.06843 | -8.13760 | -5.57050 | 0.802631 |
| 21      | 1.042336 | 38.4     | 27.2     | 0.16   | -0.3    | 0.18   | -0.20529 | -2.50110 | -2.57533 | 1.411764 |
| 22      | 1.103904 | 180.6    | 95.6     | 1.42   | -0.56   | 0.5    | -0.57886 | -11.5518 | -3.50724 | 1.889121 |
| 23      | 7.2844   | 25       | 14.5     | 0.11   | -0.1    | 0.0645 | -0.24744 | -3.70497 | -3.45045 | 1.724137 |
| 24      | 1.301639 | 34.8     | 30       | 0.16   | -0.24   | 0.18   | -0.27980 | -4.31125 | -2.98571 | 0.675734 |
| 25      | 0.835849 | 193.3    | 146.4    | 1.46   | -0.45   | 0.47   | -0.56159 | 47.47682 | -5.42382 | 0.714285 |
| 26      | 0.658468 | 20.9     | 9        | 0.1    | -0.1    | 0.075  | -0.30794 | -3.38410 | -2.42971 | 0.579310 |
| 27      | 1.087316 | 35.2     | 37       | 0.21   | -0.28   | 0.27   | -0.44497 | -25.5033 | -2.87018 | 0.951351 |
| 28      | 0.630157 | 137.8    | 164.8    | 0.8    | 0.75    | -0.78  | -0.42963 | -37.3796 | 1.994405 | 0.857142 |

TOP LAYER SHEARED OFF  
REPEAT OF 18

TABLE 10

M.V. Arctic 1.2.3 Worksheet for Test No. 3848 (Impulse/Floe)  
(Impulse Phase)

| TEST NO. 3848 | M.V. ARCTIC | IMPULSE | NEAR A FLOE | NOV. 27, 1986 | 20 Hz Sampling Rate |       |           |        |        |          |          |
|---------------|-------------|---------|-------------|---------------|---------------------|-------|-----------|--------|--------|----------|----------|
| RUN NO.       | TYPE        | TIME(s) | RATE        | h (m)         | Bp (m)              | x (m) | Fmax(kgf) | 2008   | 200S   | FM12     | FMIP     |
| 1             | SURGE       | 14.75   | F           | 15            | 30                  | 20    | 105.8     | 0.3    |        | 0.167770 |          |
| 2             | PITCH       | 9.9     | F           |               |                     |       | 54.8      | 0.825  | -0.6   | 0.612950 | 0.069080 |
| 3             | HEAVE       | 7.8     |             | 15            |                     |       |           |        |        |          |          |
| 4             | PITCH       | 10.05   | F           | 6             |                     |       | 56        | 0.8    | -0.575 | 0.646270 | 0.132218 |
| 5             | HEAVE       | 11.1    | F           |               |                     |       |           |        |        |          |          |
| 6             | SURGE       | 16.1    | F           |               |                     |       | 44        |        |        |          |          |
| 7             |             | 17.3    | F           |               |                     |       | 63        | 0.3    |        | -0.30463 |          |
| 8             | SURGE       | 20.9    |             | 10            | 30                  | 20    | 81.4      | 0.23   |        | 0.171897 |          |
| 9             | HEAVE       | 9.5     |             |               |                     |       |           |        |        |          |          |
| 10            | SURGE       | 24.1    | S           |               |                     |       | 25.6      | 0.7    |        | -0.87890 |          |
| 11            | HEAVE       | 10.45   | S           |               | 30                  | 20    |           |        |        |          |          |
| 12            | SURGE       | 28.6    | F           |               | 30                  | 30    | 89.1      | 0.36   |        | -0.18046 |          |
| 13            | HEAVE       | 9.45    |             |               | 30                  |       |           |        |        |          |          |
| 14            | SURGE       | 21.2    |             |               | 12                  | 20    | 81.9      | 0.25   |        | 0.084768 |          |
| 15            | HEAVE       | 10.6    |             | 10            | 12                  |       |           |        |        |          |          |
| 16            | SURGE       | 26.7    |             | 10            | 50                  |       | 63.6      |        |        |          |          |
| 17            | HEAVE       | 8.85    | F           | 10            | 50                  | 20    |           | 0.2125 |        | -0.00896 |          |
| 18            | PITCH       | 9.9     |             | 10            |                     |       | 48.7      | 0.8125 | -0.596 | 0.469683 | -0.03879 |
| 19            |             | 8.1     |             | 10            | 12                  |       | 45.7      | 0.7    | -0.495 | 0.476336 | 0.063146 |
| 20            |             | 8.7     | F           |               | 30                  |       | 50.8      | 0.795  | -0.568 | 0.482043 | 0.036126 |
| 21            |             | 9.55    | S           |               | 30                  | 20    | 30.9      | 0.471  | -0.32  | 0.356203 | 0.085992 |
| 22            |             | 9.2     | F           | 10            | 30                  | 30    | 49.8      | 0.85   | -0.62  | 0.433918 | -0.05820 |
| 23            |             | 8.45    |             |               | 30                  | 10    | 52.2      | 0.625  | -0.45  | 0.975402 | 0.349916 |
| 24            | HEAVE       | 9.7     |             |               | 30                  |       |           |        |        |          |          |

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TABLE 10 (Continued)  
(Oscillation Phase)

TEST NO. 3848

| RUN NO. | TYPE  | D1     | D2      | Td    | DAMPING  | AM Coef. | DP1    | DP2   | TdP   | DAMP     | RMP               |
|---------|-------|--------|---------|-------|----------|----------|--------|-------|-------|----------|-------------------|
| 1       | SURGE | 19.05  | 16.5    | 4.145 | 0.022856 | 0.708286 |        |       |       |          |                   |
| 2       | PITCH | 3      | 1.95    | 1.045 | 0.068373 | 0.548568 |        | 3.5   | 1.5   | 0.845    | 0.135569 0.194269 |
| 3       | HEAVE | 2.17   | 0.9     | 1.167 | 0.138661 | 0.903018 |        |       |       |          |                   |
| 4       | PITCH | 2.88   | 2.1     | 1.022 | 0.050186 | 0.484360 |        | 3.35  | 1     | 0.867    | 0.188872 0.234466 |
| 5       | HEAVE | 3.6    | 2       | 1.684 | 0.093105 | 2.863871 |        | 0.5   | 0.35  | 1.028    | 0.056652 0.793935 |
| 6       | SURGE | 6.75   | 4.5     | 4.12  | 0.064372 | 0.681626 |        |       |       |          |                   |
| 7       |       | 12.6   | 6       | 3.63  | 0.117221 | 0.292834 |        |       |       |          |                   |
| 8       | SURGE | 15     | 12.5    | 4.19  | 0.028993 | 0.745023 |        |       |       |          |                   |
| 9       | HEAVE |        |         |       |          |          |        |       |       |          |                   |
| 10      | SURGE | 10.625 | 8.125   | 4.134 | 0.042639 | 0.697028 |        |       |       |          |                   |
| 11      | HEAVE | 2.7    | 1.5     | 1.162 | 0.093105 | 0.907058 | 0.336  | 0.294 | 0.983 | 0.021238 | 0.644853          |
| 12      | SURGE | 19.05  | 16.5    | 4.12  | 0.022856 | 0.687742 |        |       |       |          |                   |
| 13      | HEAVE | 2.4    | 0.8     | 1.207 | 0.172169 | 1.014091 | 0.6875 | 0.475 | 0.97  | 0.058721 | 0.596835          |
| 14      | SURGE | 15.875 | 11.25   | 4.2   | 0.054705 | 0.749586 |        |       |       |          |                   |
| 15      | HEAVE | 2.7    | 0.51    | 1.25  | 0.255284 | 1.079924 | 0.615  | 0.375 | 0.983 | 0.078459 | 0.635468          |
| 16      | SURGE | 19.125 | 16.5625 | 4.134 | 0.022860 | 0.699229 |        |       |       |          |                   |
| 17      | HEAVE | 2.75   | 1.25    | 1.207 | 0.124461 | 1.043465 |        |       |       |          |                   |
| 18      | PITCH | 3.2    | 1.6     | 1.082 | 0.109508 | 0.647927 |        |       |       |          |                   |
| 19      |       | 2.7    | 1       | 1.09  | 0.155080 | 0.651485 |        |       |       |          |                   |
| 20      |       | 2.8    | 1.5     | 1.03  | 0.098811 | 0.555429 | 4.2    | 1.2   | 0.894 | 0.198459 | 0.309105          |
| 21      |       | 2.25   | 1.62    | 1.073 | 0.052190 | 0.635865 | 3.68   | 0.8   | 0.838 | 0.235928 | 0.129359          |
| 22      |       | 2.8    | 1.8     | 1.073 | 0.070118 | 0.632269 | 5      | 1.25  | 0.96  | 0.215371 | 0.496692          |
| 23      |       | 2.25   | 1.5     | 1.039 | 0.064372 | 0.531653 | 2.75   | 1     | 0.885 | 0.158891 | 0.300163          |
| 24      | HEAVE | 3.92   | 2.08    | 1.207 | 0.081    | 132.9758 | 4.75   | 1.375 | 0.97  | 0.193496 | 0.542366          |
|         |       |        |         |       |          |          | 3.25   | 1     | 0.94  | 0.184301 | 0.453565          |
|         |       |        |         |       |          |          | 0.81   | 0.58  | 0.89  | 0.053062 | 0.345154          |

TABLE II  
M.V. Kigoriak I.2.3 Worksheet for Test No. 3859 (Impact/9.3 MPa)  
(Impact Phase)

| M.V. KIGORINK IMPACT DEC. 08, 1986 50 Hz |         |         |          |         |       |        |         |         |        |        |         |          |
|------------------------------------------|---------|---------|----------|---------|-------|--------|---------|---------|--------|--------|---------|----------|
| TEST NO. 3859                            | RUN NO. | TIME(s) | SIG(MPa) | Hica(m) | Bp(m) | V (Kn) | Fx(kgf) | Fz(kgf) | D1I    | D2I    | D3I     | AMHI     |
|                                          | 1       | 5.56    | 9.3      | 15      | 40    | 5      | 34      | 101.8   | 0.0995 | 0.3    | -0.225  | -0.42762 |
|                                          | 2       | 4.02    |          |         |       | 10     | 110     | 289.2   | 0.385  | 1.0625 | -1      | -0.52141 |
|                                          | 3       | 4.44    |          |         |       | 15     | 228     | 446.7   | 0.97   | 1.65   | -1.875  | -0.60627 |
|                                          | 4       | 6.44    |          |         | 25    | 5      | 63.5    | 141.6   | 0.254  | 0.425  | -0.3875 | -0.58123 |
|                                          | 5       | 6       |          |         |       | 10     | 162.5   | 340.3   | 0.381  | 1.0625 | -0.88   | -0.28557 |
|                                          | 6       | 4.48    |          |         |       | 15     | 280     | 540.6   | 0.725  | 1.6    | -1.4    | -0.35308 |
|                                          | 7       | 5.42    |          |         | 12    | 5      | 54.6    | 113.1   | 0.013  | 0.45   | -0.225  | 6.035175 |
|                                          | 8       | 5.16    |          |         |       | 10     | 162.5   | 296.7   | 0.8314 | 0.75   | -1      | -0.67260 |
|                                          | 9       | 3.96    |          |         |       | 15     | 220     | 408     | 0.763  | 1.094  | -1.031  | -0.51702 |
|                                          | 10      | 6.4     |          |         | 10    | 5      | 66.1    | 110.6   | 0.327  | 0.32   | -0.231  | -0.66140 |
|                                          | 11      | 6.34    |          |         |       | 10     | 193.4   | 346.5   | 0.466  | 1.094  | -0.835  | -0.30482 |
|                                          | 12      | 4.3     |          |         |       | 15     | 330     | 469.3   | 0.975  | 1.65   | -1.312  | -0.43306 |
|                                          | 13      | 4.8     |          |         | 25    | 5      | 66.13   | 132.3   | 0.107  | 0.35   | -0.25   | 0.035238 |
|                                          | 14      | 5.7     |          |         |       | 10     | 173.9   | 333.5   | 0.436  | 0.6    | -0.85   | -0.33190 |
|                                          | 15      | 6.04    |          |         |       | 15     | 345     | 449.4   | 1.41   | 1.3    | -2.062  | -0.59014 |
|                                          | 16      | 4.88    |          |         | 10    | 5      | 69.2    | 125.7   | 0.182  | 0.313  | -0.32   | -0.36311 |
|                                          | 17      | 6.68    |          |         |       | 5      | 177.9   | 281.9   | 0.731  | 0.975  | -0.8    | -0.59235 |
|                                          | 18      | 5       |          |         |       | 10     | 282.5   | 379.3   | 1.188  | 1.3375 | -1.275  | -0.60168 |
|                                          | 19      | 5.84    |          |         |       | 15     | 67.3    | 112.1   | 0.172  | 0.32   | -0.35   | -0.34459 |
|                                          | 20      | 5.16    |          |         | 5     | 5      | 170     | 258     | 0.781  | 1.1    | -0.97   | -0.63539 |
|                                          | 21      | 3.62    |          |         |       | 10     | 300     | 375     | 1.41   | 1.6    | -1.5    | -0.64360 |
|                                          | 22      | 6.16    |          |         |       | 15     | 60.8    | 113     | 0.2    | 0.4375 | -0.34   | -0.49078 |
|                                          | 23      | 5.54    |          |         | 25    | 5      | 197.9   | 297.2   | 0.806  | 0.93   | -0.925  | -0.58872 |
|                                          | 24      | 4.06    |          |         |       | 10     | 420     | 534     | 0.913  | 2.25   | -1.5    | -0.22944 |
|                                          | 25      | 4.78    |          |         |       | 15     | 77.7    | 138.8   | 0.149  | 0.486  | -0.3    | -0.12650 |
|                                          | 26      | 5.42    |          |         | 12    | 5      | 170     | 240     | 0.676  | 1      | -1.06   | -0.57876 |
|                                          | 27      | 6.02    |          |         |       | 10     | 300     | 432     | 1.038  | 1.2    | -1.275  | -0.51588 |
|                                          | 28      | 6.04    |          |         |       | 15     | 45.1    | 105.8   | 0.114  | 0.368  | -0.285  | -0.33732 |
|                                          | 29      | 4.48    |          |         | 12    | 5      | 120     | 289.6   | 0.166  | 1      | -0.925  | 0.210873 |
|                                          | 30      | 4.2     |          |         |       | 10     | 160     | 330     | 0.693  | 1.4    | -1.35   | -0.61326 |
|                                          | 31      | 4.22    |          |         |       | 15     |         |         |        |        |         |          |

TABLE 11 (Continued)  
(Beaching Phase)

TEST NO. 3859

| RUN NO. | RMZ1     | RMZ2     | RMZ3 | Fxb(kgf) | Fzb(kgf) | D1B    | D2B     | D3B    | RMZ8     | RMZP     | RMZB     | Fx/Fz                |
|---------|----------|----------|------|----------|----------|--------|---------|--------|----------|----------|----------|----------------------|
| 1       | 2.828882 | 0.958438 |      | 22       | 52.5     | 0.03   | 0.04    | 0.016  | 0.228364 | 2.696476 | 25.30208 | 0.333988             |
| 2       | 7.226229 | 0.416203 |      | 314      | 110      | 0.285  | -0.28   | 0.25   | 0.845484 | -9.33655 | -3.09622 | 0.380359             |
| 3       | -12.4646 | 0.279906 |      | 655.8    | 157.5    | 0.64   | -1.65   | 0.375  | 0.716394 | -1.39693 | -1.78555 | 0.510409             |
| 4       | 7.002899 | 0.760196 |      | 36       | 67.5     |        |         |        |          |          |          | 0.448446SAT. 13      |
| 5       | 3.322319 | 0.797139 |      | 175      | 100      | 0.201  | -0.125  | 0.1    | 0.458369 | -11.7962 | -5.48888 | 0.477519             |
| 6       | 5.458827 | 0.82002  |      | 719.8    | 75       | 1.95   | -1.6    | 1.2    | -0.38169 | -1.52891 | -1.27053 | 0.517943             |
| 7       | 0.558659 | 0.692311 |      | 36       | 62.5     | 0.026  | -0.51   | 0.24   | 1.319288 | -1.72175 | -1.84166 | 0.482756SAT. 13, 14  |
| 8       | -5.89399 | 0.712382 |      | 195.9    | 100      |        |         |        |          |          |          | 0.547691             |
| 9       | 10.39503 | 0.9392   |      | 374.5    | 180      | 1.426  | -0.875  | 0.812  | -0.56009 | -6.57255 | -2.07765 | 0.539215SAT. 12      |
| 10      | 2.570691 | 1.027332 |      | 42       | 53.1     | 0.13   | -0.16   | 0.0625 | -0.45883 | -2.71935 | -3.41038 | 0.597649             |
| 11      | 2.736126 | 0.814230 |      | 150      | 120      | 0.263  | -0.606  | 0.05   | -0.04465 | -1.70087 | -2.84756 | 0.558152             |
| 12      | 2.766803 | 0.500246 |      | 407.2    | 170      | 0.764  | -0.45   | 0.45   | -0.10722 | -24.6117 | -2.90777 | 0.703174             |
| 13      | 2.818195 | 1.22705  |      | 44.8     | 52.5     |        |         |        |          |          |          | 0.499848             |
| 14      | -6.29152 | 1.323    |      | 135      | 104      |        |         |        |          |          |          | 0.521439             |
| 15      | -3.24070 | 0.350071 |      | 405.9    | 137.5    |        |         |        |          |          |          | 0.767690             |
| 16      |          |          |      | 52.1     | 50       |        |         |        |          |          |          | 1.042SAT. 44         |
| 17      | 41.26100 | 1.005639 |      | 56       | 52.5     |        |         |        |          |          |          | 0.550517             |
| 18      | 3.243108 | 0.504050 |      | 120      | 110.25   |        |         |        |          |          |          | 0.631074REPEAT OF 16 |
| 19      | 8.559003 | 0.465384 |      | 428.2    | 120      |        |         |        |          |          |          | 0.744793SAT. 12      |
| 20      | -32.1810 | 0.689865 |      | 44       | 10.75    |        |         |        |          |          |          | 0.600356SAT. 44      |
| 21      | 3.640018 | 0.258940 |      | 177.3    | 274.5    |        |         |        |          |          |          | 0.645901             |
| 22      | 5.861925 | 0.221774 |      | 521.1    | 416.2    |        |         |        |          |          |          | 1.252042SAT. 12      |
| 23      | 2.198994 | 0.467909 |      | 70       | 113.4    |        |         |        |          |          |          | 0.617283SAT. 13      |
| 24      | 17.19726 | 0.618177 |      | 118.75   | 116      |        |         |        |          |          |          | 0.665881             |
| 25      | 1.103403 | 0.43824  |      | 518.1    | 456      |        |         |        |          |          |          | 0.786516             |
| 26      | 1.245633 | 0.783562 |      | 80       | 52.5     |        |         |        |          |          |          | 0.559798SAT. 12, 44  |
| 27      | -168.783 | 0.176499 |      | 199.6    | 242.5    |        |         |        |          |          |          | 0.823092             |
| 28      | -167.924 | 0.762909 |      | 556      | 400      |        |         |        |          |          |          | 0.694444             |
| 29      | 1.928365 | 0.587786 |      | 37       | 62.5     | 0.0113 | -0.0425 | 0.003  | 4.484650 | -6.14202 | -14.8736 | 0.426275             |
| 30      | 6.653988 | 0.519459 |      | 356.6    | 100      | 0.363  | -0.25   | 0.25   | 0.645509 | -26.0006 | -3.02    | 0.414364SMOOTH       |
| 31      | 7.937167 | 0.212    |      | 721.4    | 407.8    | 1.626  | -0.95   | 1.35   | -0.25684 | 3.037605 | -2.79077 | 1.769004SMOOTH       |

TABLE 12  
M.V. Kigoriak 1.2.3 Worksheet for Test No. 3862 (Impulse/Floe)  
(Impulse Phase)

| M.V. KIGORIAK IMPULSE NEAR A FLOE DEC. 10, 1986 |       |         |      |       |        |       |           |        |          |          |  |  | 20 Hz |
|-------------------------------------------------|-------|---------|------|-------|--------|-------|-----------|--------|----------|----------|--|--|-------|
| RUN NO.                                         | TYPE  | TIME(s) | RATE | h (m) | Bp (m) | x (m) | Fmax(kgf) | 2008   | 2005     | PM12     |  |  |       |
| 1                                               | HEAVE | 7.4 F   |      |       |        | 20    |           |        |          |          |  |  |       |
| 2                                               | PITCH | 7.06 F  |      |       |        |       | 150.4     | 0.428  | -0.378   | 5.114934 |  |  |       |
| 3                                               |       | 7.2     |      | 10    |        |       | 156.6     | 0.48   | -0.33    | 1.665114 |  |  |       |
| 4                                               |       | 8.4 S   |      |       |        |       | 77.35     | 0.26   | -0.14    | 0.982320 |  |  |       |
| 5                                               | HEAVE | 7.5 F   |      |       |        |       |           |        |          |          |  |  |       |
| 6                                               |       | 8.7 S   |      |       |        |       |           |        |          |          |  |  |       |
| 7                                               |       | 8.3 F   |      |       |        | 10    |           |        |          |          |  |  |       |
| 8                                               | PITCH | 8.95    |      |       |        |       | 143.8     | 0.74   | -0.428   | 0.591428 |  |  |       |
| 9                                               |       | 7.25    |      |       |        | 30    | 153.8     | 0.61   | -0.34    | 1.525143 |  |  |       |
| 10                                              | HEAVE | 7.3     |      |       |        |       |           |        |          |          |  |  |       |
| 11                                              |       | 7.45    |      |       | 12     | 20    |           |        |          |          |  |  |       |
| 12                                              | PITCH | 6.7     |      |       |        |       | 171.4     | 0.7    | -0.3     | 0.345371 |  |  |       |
| 13                                              |       | 7.6     |      |       | 40     |       | 149.4     | 0.8    | -0.32    | -0.01865 |  |  |       |
| 14                                              | HEAVE | 7.85    |      |       |        |       |           |        |          |          |  |  |       |
| 15                                              |       | 7.35    |      | 5     | 25     |       |           |        |          |          |  |  |       |
| 16                                              | PITCH | 6.45    |      |       |        |       | 160.7     | 0.68   | -0.31    | 0.567667 |  |  |       |
| 17                                              | SURGE | 10.35 F |      | 5     | 25     | 20    | 150       | 0.3    | -0.16247 |          |  |  |       |
| 18                                              |       | 11.2    |      | 15    |        |       | 171.65    | 0.29   | -0.00854 |          |  |  |       |
| 19                                              |       | 12.1    |      | 10    |        |       | 156.7     | 0.255  | 0.029329 |          |  |  |       |
| 20                                              |       | 14.1 S  |      |       |        |       | 87        | 0.18   | -0.19059 |          |  |  |       |
| 21                                              |       | 13.9 F  |      |       |        | 10    | 158.5     | 0.2625 | 0.011406 |          |  |  |       |
| 22                                              |       | 13.75   |      |       |        | 30    | 150.4     | 0.4125 | -0.36926 |          |  |  |       |
| 23                                              |       | 15.1    |      |       | 40     | 20    | 155.1     | 0.2375 | 0.093890 |          |  |  |       |
| 24                                              |       | 13.35   |      |       | 12     |       | 162.9     | 0.28   | -0.02848 |          |  |  |       |

TABLE 12 (Continued)  
(Oscillation Phase)

| TEST NO. 3962 | RUN NO. | TYPE  | AMIP     | D1     | D2    | Td   | DAMPING  | AM Coef. | DP1  | DP2  | Tdp   | DAMP     | AMP      |
|---------------|---------|-------|----------|--------|-------|------|----------|----------|------|------|-------|----------|----------|
|               | 1       | HEAVE |          | 5.61   | 2.46  | 1.39 | 0.130039 | 1.153279 | 1    | 0.75 | 1.467 | 0.045719 | 1.772666 |
|               | 2       | PITCH | 0.6463   | 2.15   | 1.17  | 1.39 | 0.096350 | 1.169984 | 9.76 | 2.44 | 1.075 | 0.215371 | 0.422775 |
|               | 3       |       | 0.703283 | 2.34   | 1.31  | 1.39 | 0.091901 | 1.171819 | 5.85 | 0.63 | 1.49  | 0.334151 | 1.546238 |
|               | 4       |       | 0.953087 | 0.164  | 0.086 | 1.4  | 0.168501 | 1.158860 | 2.4  | 0.8  | 1.37  | 0.172169 | 1.351359 |
|               | 5       | HEAVE |          | 5.75   | 2.25  | 1.38 | 0.147634 | 1.111860 | 2.5  | 0.6  | 1.37  | 0.221406 | 1.304401 |
|               | 6       |       |          | 2.75   | 1.5   | 1.49 | 0.095985 | 1.493620 | 1.6  | 0.6  | 1.5   | 0.154175 | 1.835833 |
|               | 7       |       |          | 5.365  | 2.54  | 1.33 | 0.118124 | 0.977326 | 1.8  | 0.9  | 1.17  | 0.109608 | 0.746097 |
|               | 8       | PITCH | 0.246678 | 2.15   | 1.23  | 1.29 | 0.088496 | 0.871726 | 6.77 | 2.13 | 1.33  | 0.180933 | 1.208991 |
|               | 9       |       | 0.589856 | 2.24   | 1.17  | 1.34 | 0.102777 | 1.014073 | 2.93 | 0.73 | 1.36  | 0.215877 | 1.276657 |
|               | 10      | HEAVE |          | 5.37   | 1.95  | 1.34 | 0.159105 | 0.984045 | 1.1  | 0.6  | 1.47  | 0.095985 | 1.764145 |
|               | 11      |       |          | 6.05   | 3.415 | 1.42 | 0.090606 | 1.267118 | 1.5  | 0.6  | 1.37  | 0.144248 | 1.372767 |
|               | 12      | PITCH | 0.73114  | 2.2    | 1.36  | 1.42 | 0.076295 | 1.272578 | 3.12 | 0.78 | 1.41  | 0.215371 | 1.447696 |
|               | 13      |       | 0.347267 | 2.05   | 1.025 | 1.36 | 0.109608 | 1.071601 | 2.44 | 0.5  | 1.525 | 0.244526 | 1.822988 |
|               | 14      | HEAVE |          | 6.44   | 2.2   | 1.37 | 0.168433 | 1.067377 | 1.6  | 0.48 | 1.33  | 0.188121 | 1.202933 |
|               | 15      |       |          | 5.85   | 2.44  | 1.36 | 0.137789 | 1.056983 | 1.05 | 0.36 | 1.53  | 0.167880 | 1.937061 |
|               | 16      | PITCH | 0.639464 | 2.05   | 1.17  | 1.38 | 0.088870 | 1.141865 | 2.93 | 0.68 | 1.35  | 0.226347 | 1.232405 |
|               | 17      | SURGE |          | 15.58  | 13.53 | 3.6  | 0.022438 | 0.303701 |      |      |       |          |          |
|               | 18      |       |          | 15.68  | 13.53 | 3.64 | 0.023455 | 0.332770 |      |      |       |          |          |
|               | 19      |       |          | 14.15  | 12.2  | 3.65 | 0.023583 | 0.340095 |      |      |       |          |          |
|               | 20      |       |          | 11.4   | 9.75  | 3.63 | 0.024865 | 0.325367 |      |      |       |          |          |
|               | 21      |       |          | 14.25  | 12.2  | 3.61 | 0.024702 | 0.310813 |      |      |       |          |          |
|               | 22      |       |          | 14.14  | 11.89 | 3.65 | 0.027561 | 0.339823 |      |      |       |          |          |
|               | 23      |       |          | 18.45  | 16.2  | 3.61 | 0.020685 | 0.311053 |      |      |       |          |          |
|               | 24      |       |          | 17.015 | 14.45 | 3.67 | 0.025986 | 0.354660 |      |      |       |          |          |

The surge, heave and pitch added mass coefficients are denoted AMX, AMZ and AMP, respectively. It should be mentioned that the values entered in the table for  $F_x$ ,  $F_z$ , D1, D2 and D3 were the maximum values during the impact phase and close to the time of the peak of the beaching force. A sample data set for impact Runs No. 2, 3 and 4 are shown in Figs. 44, 45 and 46.

As an example, for impact test data analysis, consider the calculations associated with run No. 7 of Test No. 3856 of Table 9. The monitoring time for this run was 11.46 s, the simulated ice strength, ice thickness, bottom proximity, and ramming speed were 14 MPa, 15 m, 12 m, and 5 knots, respectively. The peak impact horizontal and vertical forces measured were 20.6 and 25.8 kgf. The surge, bow heave, and stern vertical accelerations measured were 0.137, 0.28 and - 0.201 g's. The surge, heave and pitch added mass coefficients were calculated using Eqs. 38, 39 and 40 of Section 2.4.5 as follows:

$$\lambda_x = AMXI = \frac{20.6}{302 \times 0.137} - 1 = -0.502$$

$$\lambda_z = AMZI = \frac{25.8}{302 \left\{ -0.201 + \frac{1}{2} (0.28 + 0.201) \right\}} - 1 = 1.1628$$

$$\lambda_\theta = AMPI = \frac{25.8 \times 2.055}{304 \left( \frac{0.28 + 0.201}{4.11} \right)} - 1 = 0.4911$$

The same relationships are used to calculate the added mass values for the beaching phase.

Similarly, as an example of an impulse test data analysis, consider run No. 4 (pitch impulse) of test No. 3848 of the M.V. Arctic, Table 10. The maximum pulling force at the bow in this test was 56 kgf. The bow vertical acceleration and the stern vertical accelerations were 0.8 and - 0.575 g's. The added mass coefficients were



calculated using the same equations used in the ramming case, Eqs. 39 and 40, as follows:

$$\lambda_z = \text{AMIZ} = \frac{56}{302 \left\{ -0.575 + \frac{1}{2} (0.8 + 0.575) \right\}} - 1 = 0.648$$

$$\lambda_\theta = \text{AMIP} = \frac{56 \times 2.0555}{304 \left( \frac{0.8 + 0.201}{4.11} \right)} - 1 = 0.132$$

Subsequent to the impulse phase, a free oscillations phase takes place. The C.G. heave displacement and the pitch angle measurements are used to calculate the damping ratio and the heave/pitch added mass coefficient by applying Eqs. 32 to 36 as follows:

(a) C.G. Heave:

$$\xi = \frac{C}{C_{cr}} = \left\{ 1 + \frac{4\pi^2}{(\ln \frac{2.88}{2.1})^2} \right\}^{-1/2} = 0.05$$

$$\lambda_z = \frac{17,000}{302 \left\{ \frac{\ln(2.88/2.1)}{\xi \times 1.022} \right\}^2} - 1 = 0.484$$

(b) Pitch:

$$\xi = \left\{ 1 + \frac{4\pi^2}{(\ln \frac{3.35}{1})^2} \right\}^{-1/2} = 0.189$$

$$\lambda_z = \frac{20,455}{302 \left\{ \frac{\ln(3.35/1)}{0.189 \times 0.867} \right\}^2} - 1 = 0.234$$

The same procedures were employed in calculating Tables 11 and 12 for the M.V. Kigoriak.



The ramming test results given in Table 9 are also plotted in Figures 47, 48 and 49. Figure 47 shows the M.V. Arctic added mass coefficients for the impact phase while those for the beaching phase are shown in Fig. 48. Figure 49 depicts the relationships between the horizontal and vertical force developed during the impact and beaching phases of the ramming process. The added mass coefficients are plotted against the bottom proximity as the abscissa. Figure 50 shows a scatter diagram of the relationship between  $F_x$  and  $F_z$  in the impact and beaching phases.

It should be noted that the calculated impact and beaching phase added mass coefficients are in fact effective values since the equations employed neglect to exclude the damping and drag forces and time dependency effects. The surge added mass coefficients should not be negative as shown by the results. This may be due to one or more of the following reasons:

- (a) The forces employed should be corrected for the effects of damping and water drag forces.
- (b) The dynamic effects are significant and therefore an equation in the form of  $F = ma$  (Force = mass x acceleration) may not be adequate due to the presence of viscous damping, and time dependent effects.
- (c) The measured forces may have inaccuracies due to the damping occurring through the foam used to simulate the ice or due to the dynamic magnification effect of the force block (dynamometer) itself which was statically calibrated.
- (d) Inaccuracies in the measured surge acceleration or in its calibration.

The heave added mass coefficients exhibit high degree of variability similar to that noted in Table 8 which were obtained during the analysing of the time variation of heave added mass. Again, the reasons for this variability may be the following:

- (a) The small value of the acceleration at the C.G. which is close to the point of rotation.
- (b) The contamination of the accelerometer signal with undesirable frequencies.

- (c) The inaccuracies in the measured vertical force due to the damping effect in the foam causing time lag between measured forces and response. The fact that the calibration of the dynamometer was carried out using static loads. A dynamic magnification effect may be important.

Except, for the beaching phase where the accelerations are very small, the pitch added mass coefficient appears to be the only coefficient that is consistently reasonable. This may be due to the fact that it is less sensitive to the errors involved in the measured accelerations and forces due to the larger values of the difference  $\ddot{z}_b - \ddot{z}_s$  between the bow and stern accelerations and the pitching moment  $F_z L_b$  about the C.G., compared to the values of  $\ddot{z}_G$  and  $F_z$  used in calculating the effective heave added mass coefficient. Figure 47 shows the surge, heave, and pitch added coefficients plotted against the bottom proximity. These graphs were prepared using the Lotus 1.2.3 program "print graph" and the data ranges representing the bottom proximity and the appropriate added mass coefficients from the worksheet of Table 9. Figure 48 shows plots of similar data pertaining to the beaching phase. The scatter in the results in the beaching phase is even more pronounced than that with respect to the impact phase. This may be due to the smaller significantly accelerations measured during the beaching phase, especially near the peak of the vertical beaching force. Figure 49 compares the relationship between  $F_x$  and  $F_z$  during the impact peak and beaching. It should be noted that some impacts exhibited double peaks and in such cases the second peak was included with the beaching forces. It should be noted that although the impact data exhibit one sort of relationship, two relationships are evident for the beaching phase. The two relationships indicate the significant horizontal forces arise in high speed rams due to the effect of the ice knife or stopper which is nearly vertical giving rise only to horizontal forces. The impact forces are related as

$$F_z = 1.36 F_x \quad (51)$$

and the beaching forces are related as

$$F_z = 2.36 F_x \quad ; \text{ full partial ice knife contact} \quad (52a)$$

or

$$F_z = 0.825 F_x ; \text{ for full ice knife contact} \quad (52b)$$

Employing Eq. 31, the coefficient of friction can be calculated as

$$\mu = \frac{\xi - \tan \gamma}{1 + \xi \tan \gamma} \quad (53)$$

giving a value of 0.111 for  $\xi = 0.735$  and  $\gamma = 30^\circ$ . This indicates that the friction between the wet hull and the foam; was not significantly higher than that observed in full scale between 0.05 and 0.15 in dynamic ship ice interaction.

The results from the M.V. Arctic Impulse tests are given in Table 10. After the application of an impulse pertaining to surge, heave or pitch motions, both the impulse and the free vibration phases are used to calculate the added mass coefficients. The pitch impulse gives rise to both heave and pitching oscillations due to the application of a vertical force at the bow to incite this type of motion. The force time history was measured in case of the surge and pitch impulse. In heave, the vertical force applied at the C.G. was applied by pushing the model manually into the water and therefore, the vertical force was not measured.

The worksheet for Test No. 3848 is given in Table 10. Figure 50 shows the surge, heave-pitch, and pitch added mass coefficients plotted versus the bottom proximity. In the free vibration phase following the impulse, the added mass coefficients were calculated and are plotted in Fig. 51. The results show that the heave added mass coefficients are the highest ranging between 0.9 and 1.1 while the pitch added mass coefficients show more scatter falling between 0.2 and 0.55. The coupling coefficients between surge and heave or surge and pitch were found to be very small and were therefore neglected. On the other hand, the coupling coefficients between heave and pitch were found to be significant ranging from approximately 0.35 to 0.65. The results show that the heave-on-pitch and pitch-on-heave values are comparable.

Figure 52 shows the damping ratios calculated during the free oscillations response. The results show that the damping increases as the bottom proximity decreases due to the higher resistance to ship motion in restricted waters. The figure also shows that the damping ratio is highest for the heave motion and lowest for the surge motion. The heave damping coefficient is strongly dependent on the bottom proximity and ranges between 0.1 and 0.26. The surge damping ratio was independent of the bottom proximity and ranged between 0.02 and 0.12. The pitching damping ratios are dependent on the bottom proximity in a similar way to heave. The pitching damping ratio ranged between 0.13 and 0.24. The coupling damping ratio for pitch-on-heave is significantly smaller than for heave-on-pitch. This is due to the fact that the pitching motion induced was not pure pitching and included a significant heave component as described earlier in this section.

The presentation for the results from the Kigoriak model tests follows the same lines adopted for the M.V. Arctic. These results are presented in Tables 11 and 12 and Figures 53 to 58. In addition, Figure 59 gives a more detailed analysis of a sample set of the model tests extracted from Test No. 3862 (Impulse, near Floe). The figure represents a significant finding from the work carried out in this project. This analysis carried out in order to study the dependence of added mass coefficient on the bottom proximity, distance to ice floe, and floe thickness. These results show the surge added mass coefficient is less sensitive to these parameters and is significantly smaller than the heave, pitch and heave on pitch added mass coefficients. For the range of ice thicknesses considered 5 to 15 m, the effect of ice thickness on the pitch and heave added mass coefficients is below per cent and is therefore considered insignificant. These results indicate however that both the distance to floe and bottom proximity are of a significant influence on the added mass ranging between 10 and 30 percent.

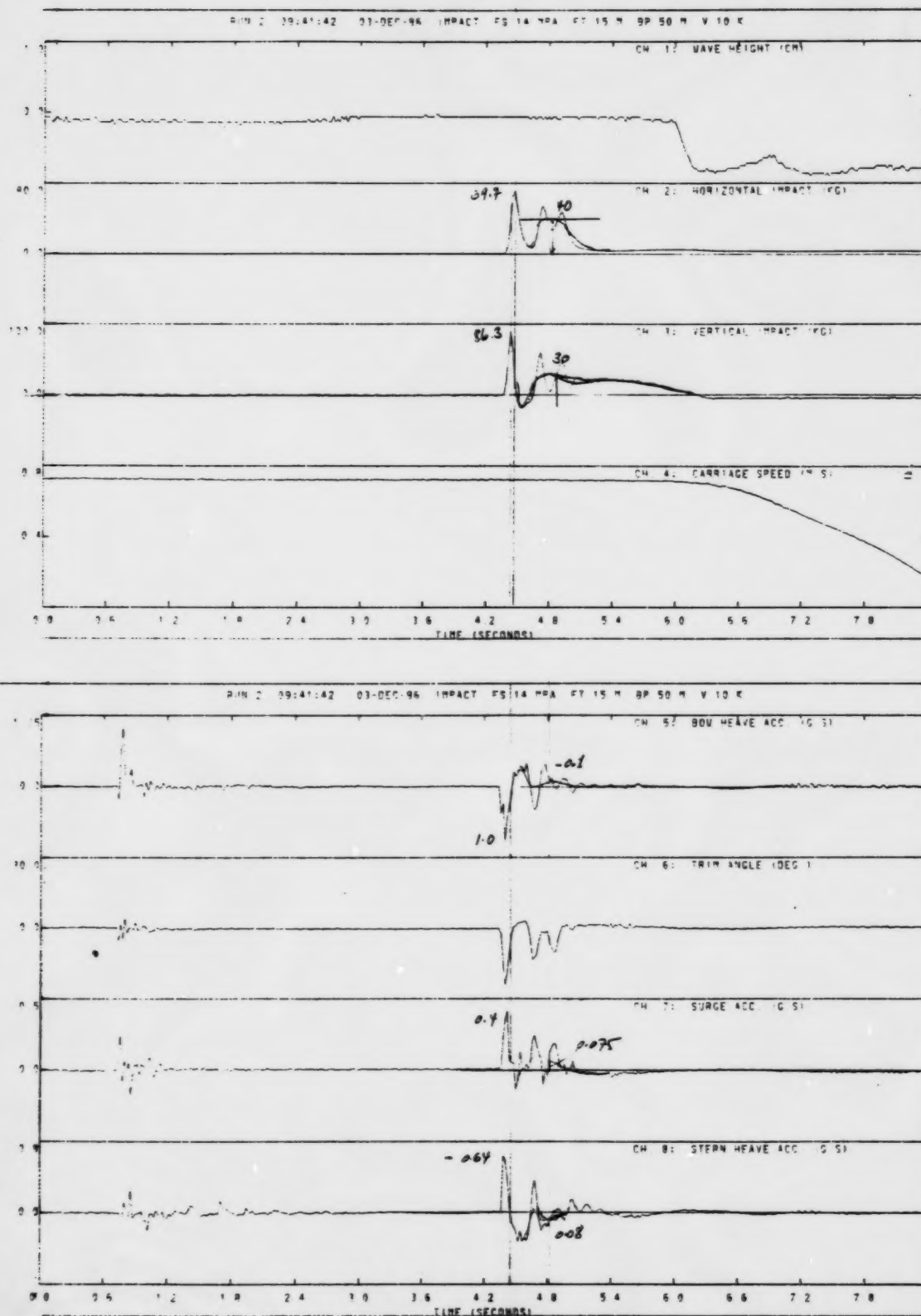


Fig. 44 Signals for Ram 2 (Test 3856)



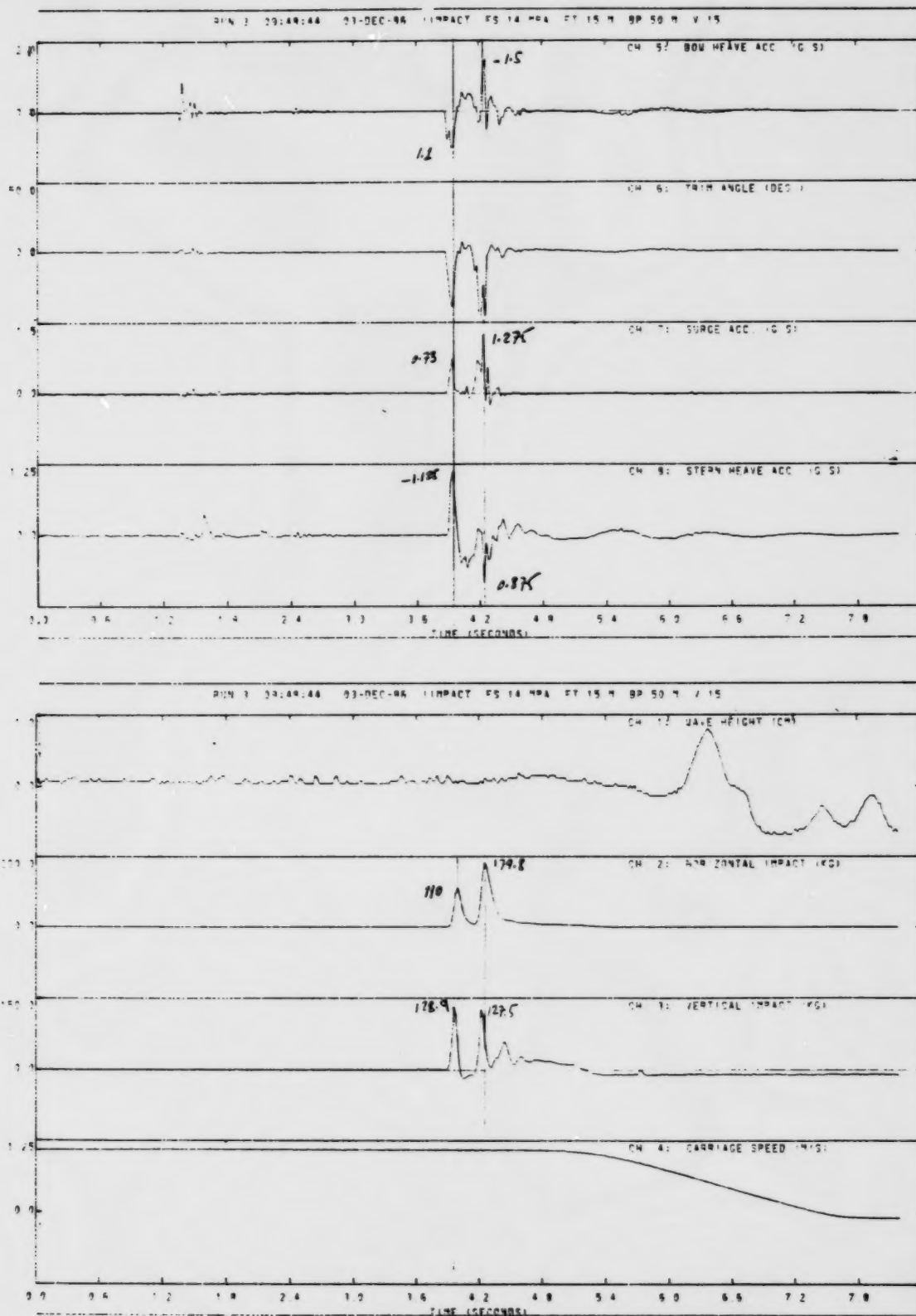


Fig. 45 Signals for Ram 3 (Test 3856)

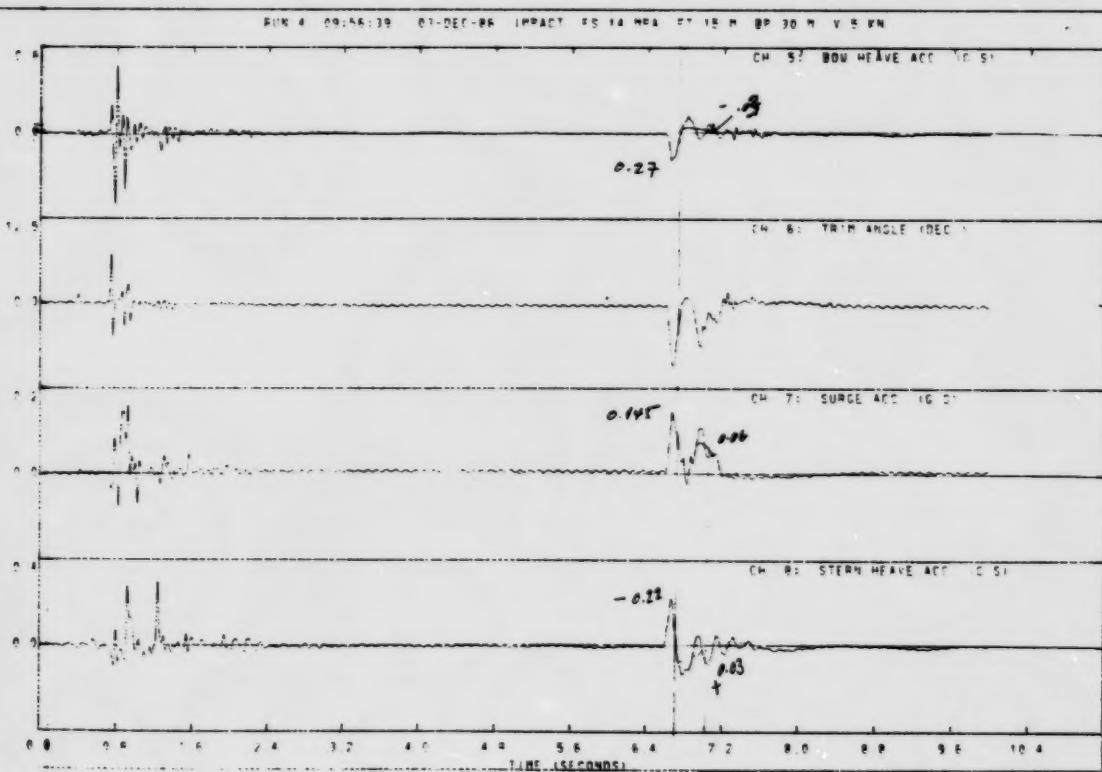
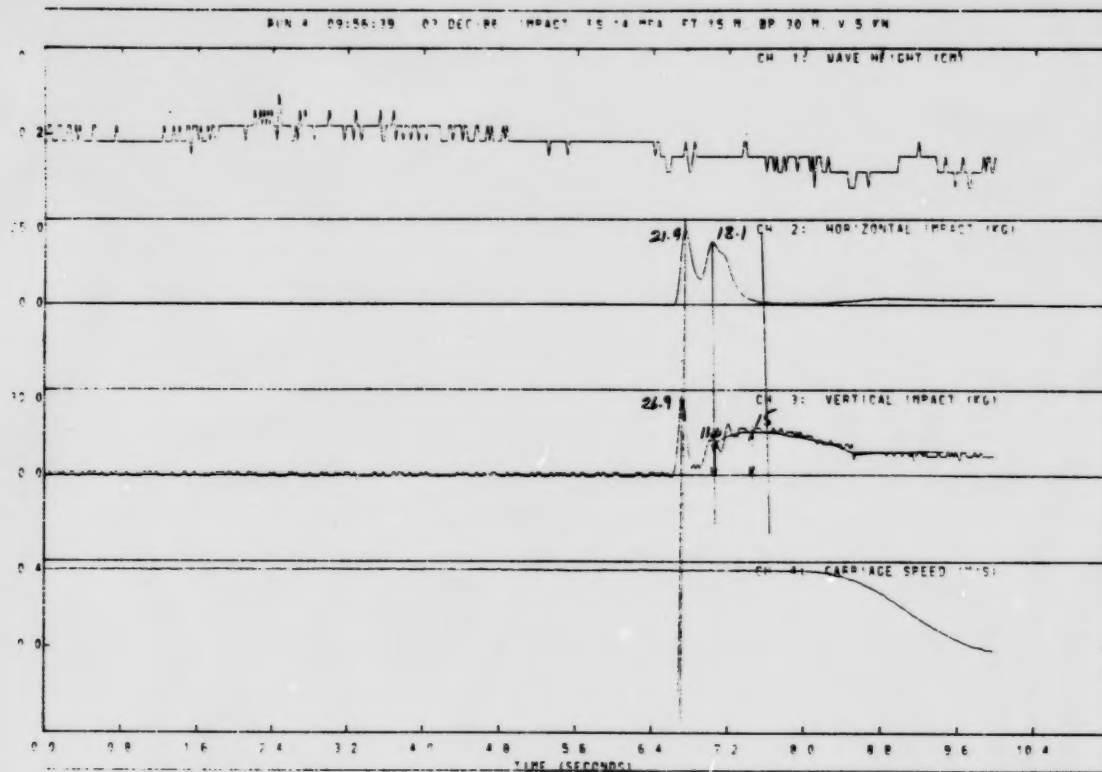
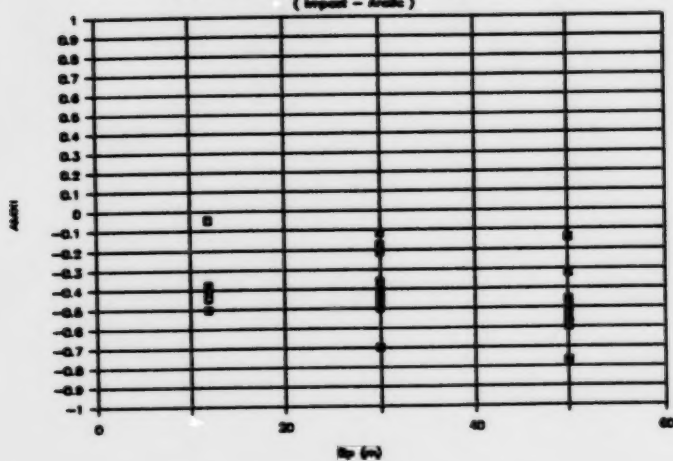


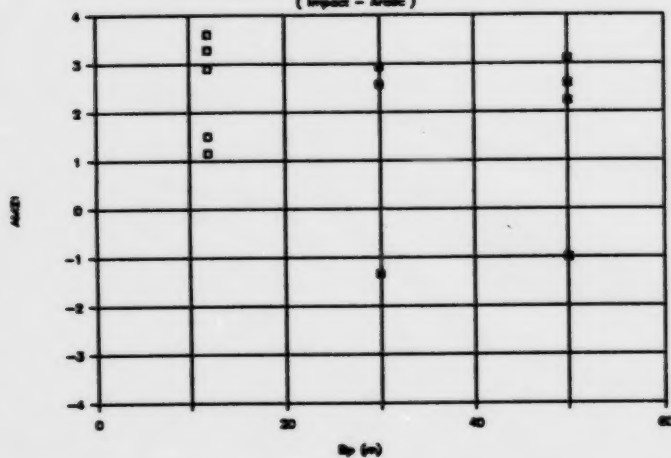
Fig. 46 Signals for Ram 4 (Test 3856)



Bottom Prox. vs Surge AMC  
( Impact - Arctic )



Bottom Prox. vs Heave AMC  
( Impact - Arctic )



Bottom Prox. vs Pitch AMC  
( Impact - Arctic )

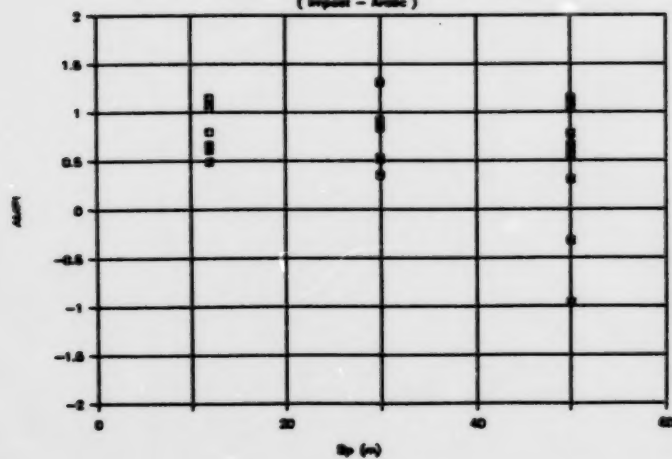


Fig. 47 Impact Added Mass Coefficients (M.V. Arctic)

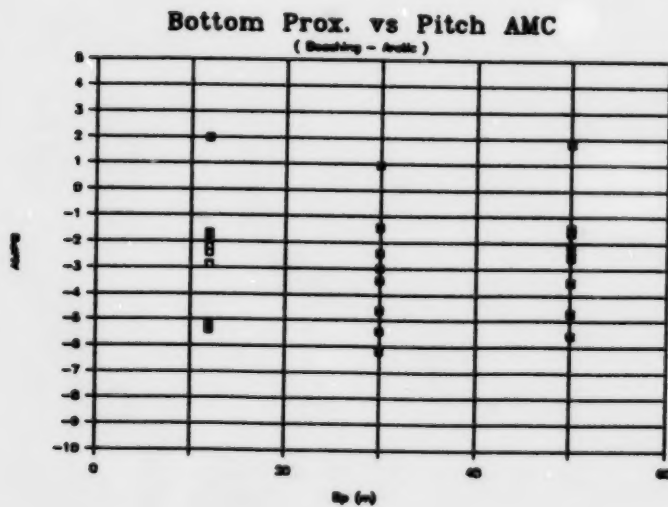
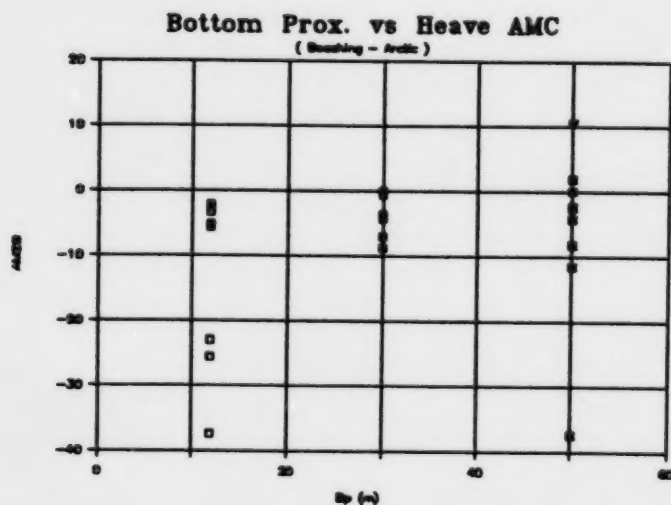
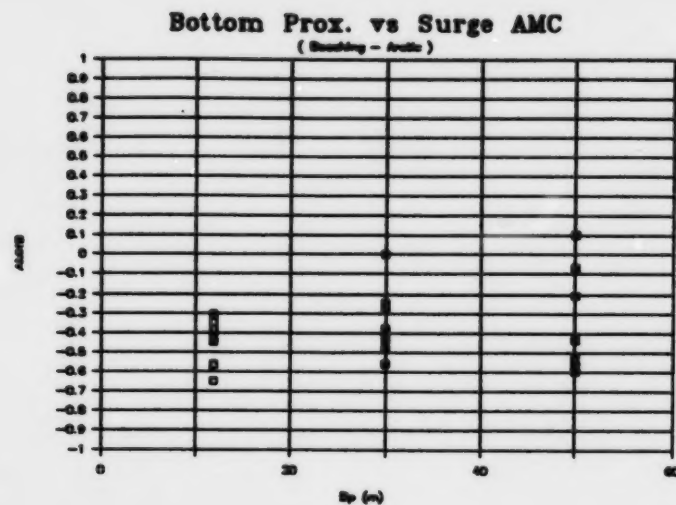
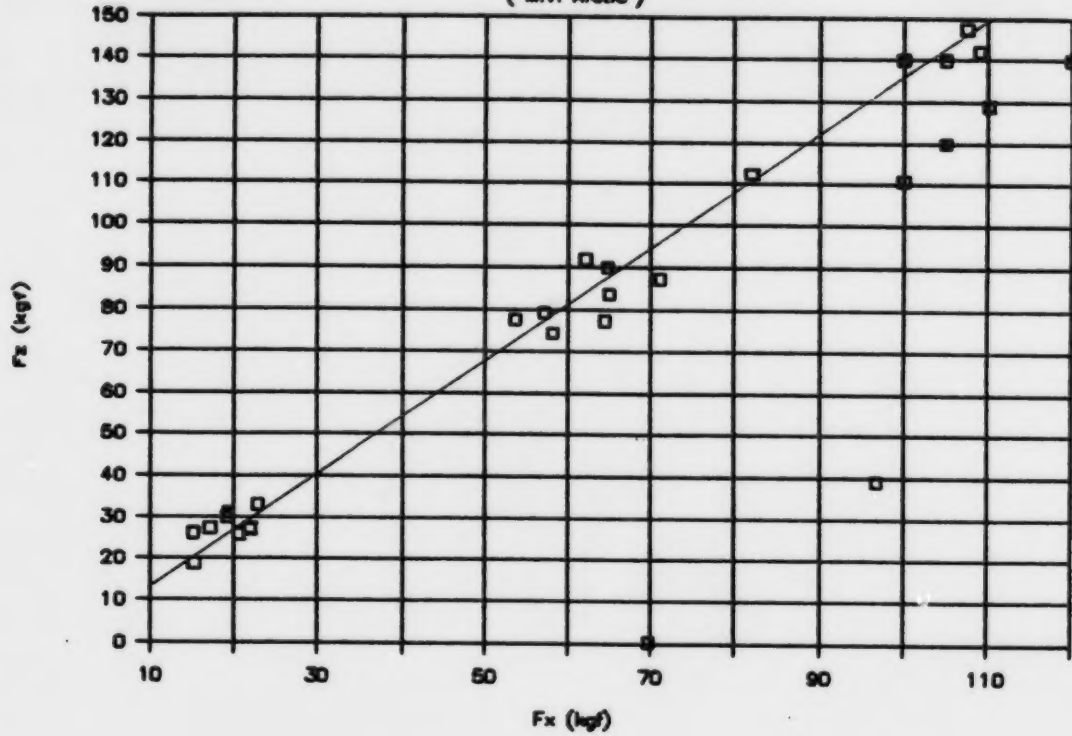


Fig. 48 Beaching Added Mass Coefficients (M.V. Arctic)

# **F<sub>x</sub> vs F<sub>z</sub> AT IMPACT**

( M.V. Arctic )



# **F<sub>x</sub> vs F<sub>z</sub> AT Beaching**

( M.V. Arctic )

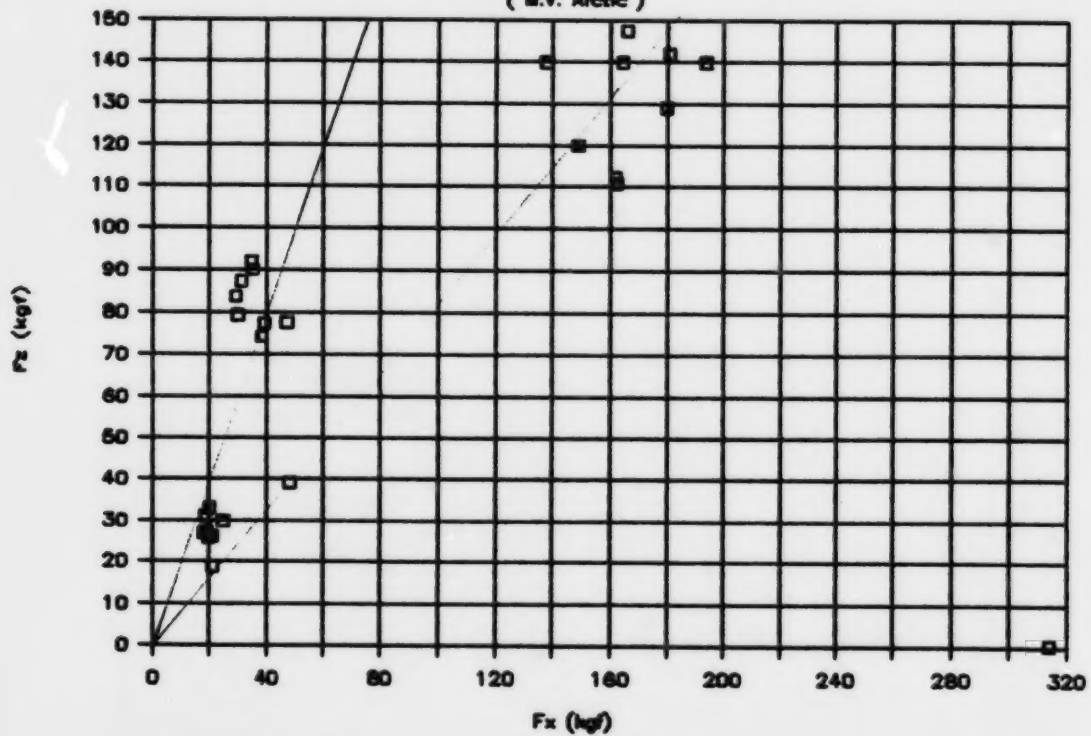
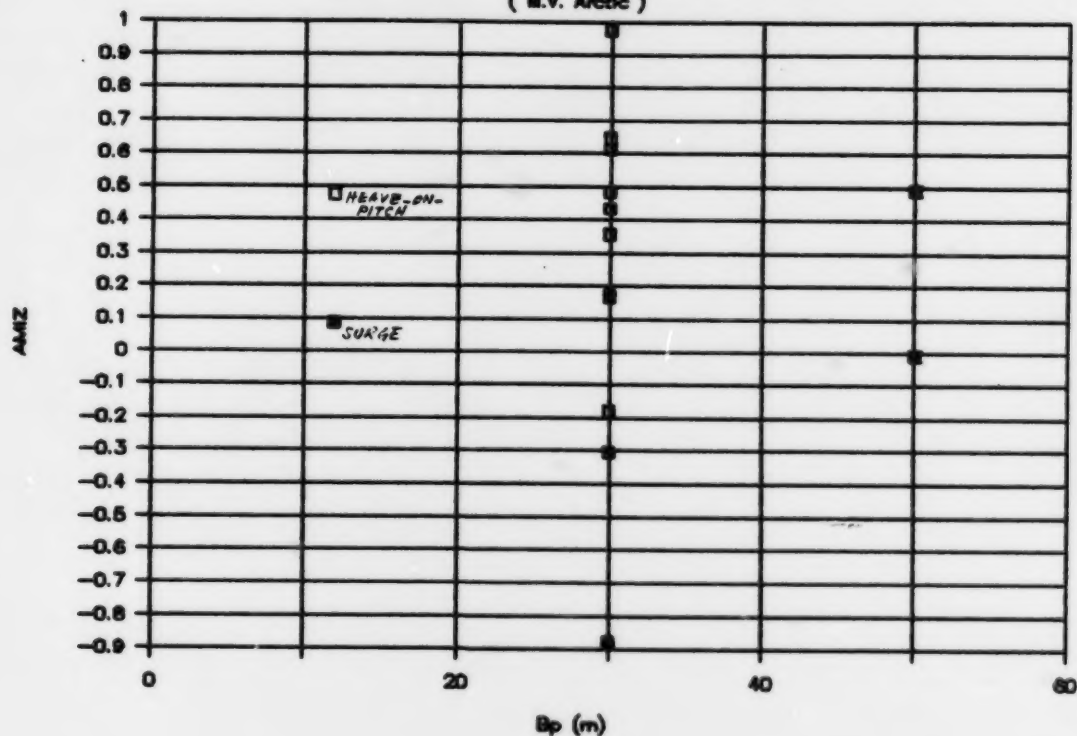


Fig. 49 Relationship Between Vertical and Horizontal Forces (M.V. Arctic)

# Bp vs. AMIZ IMPULSE

( M.V. Arctic )



# Bp vs. AMIP IMPULSE

( M.V. Arctic )

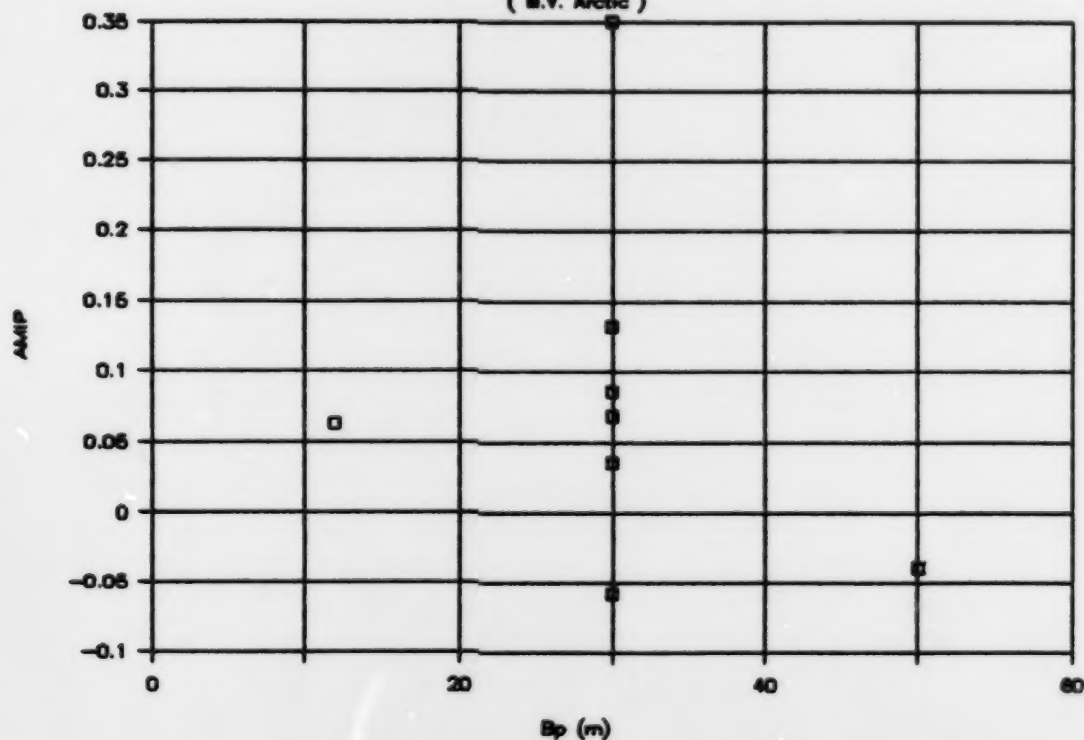
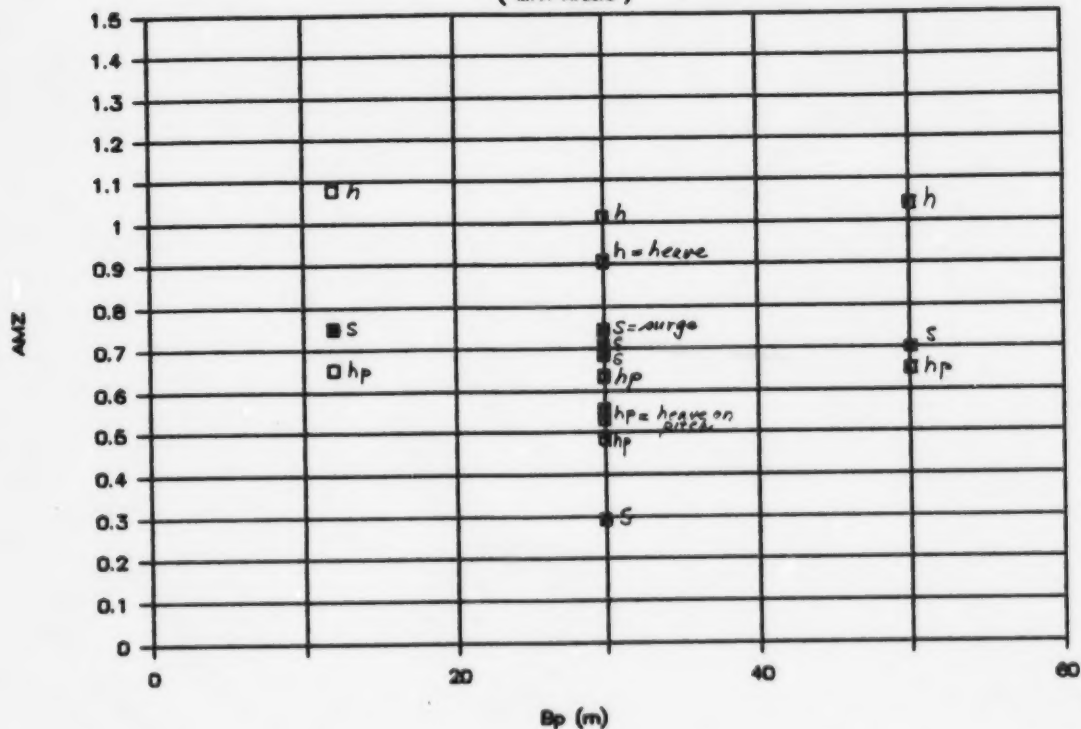


Fig. 50 Added Mass Coefficients at Impulse Peak Force (M.V. Arctic)

# Bp vs. AMZ FREE OSC.

( M.V. Arctic )



# Bp vs. AMP FREE OSC.

( M.V. Arctic )

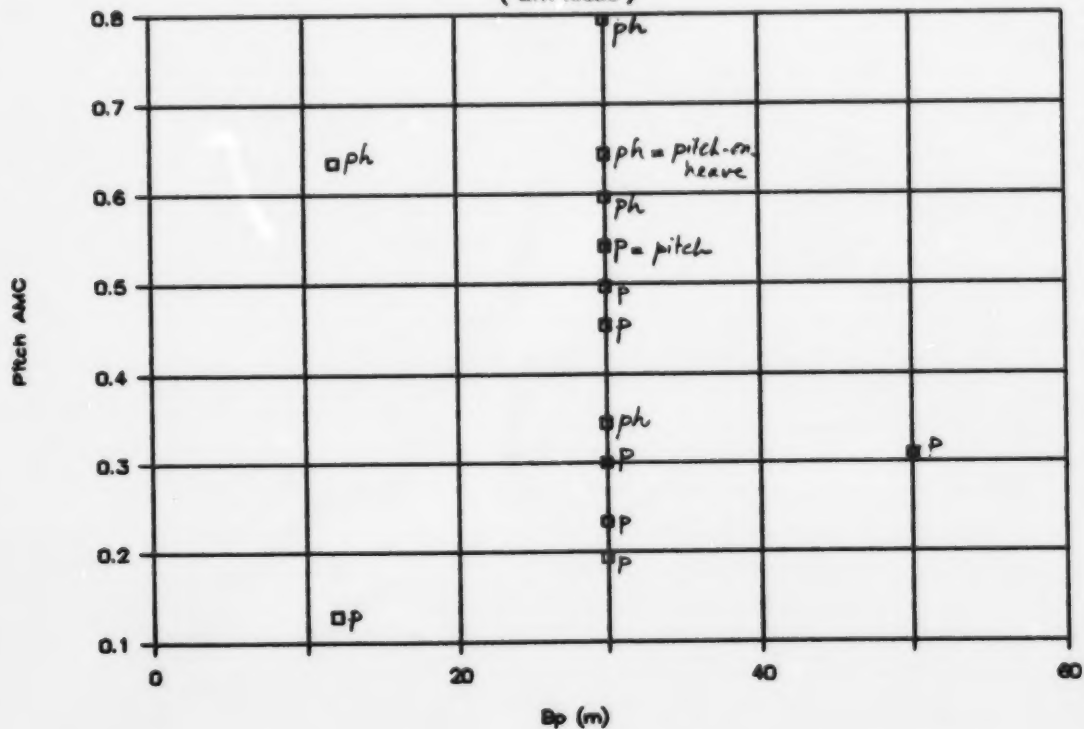
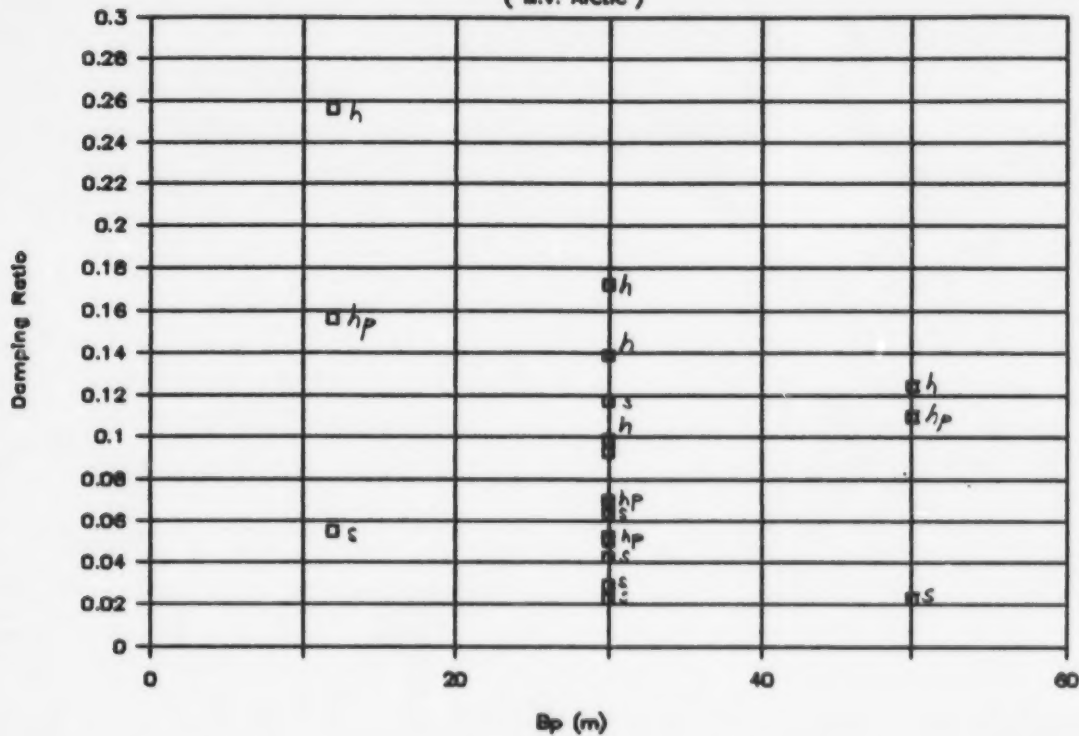


Fig. 51 Added Mass Coefficients During Free Oscillations (M.V. Arctic)

# Bp vs. Damping Ratio

( M.V. Arctic )



# Bp vs. DAMPP FREE OSC.

( M.V. Arctic )

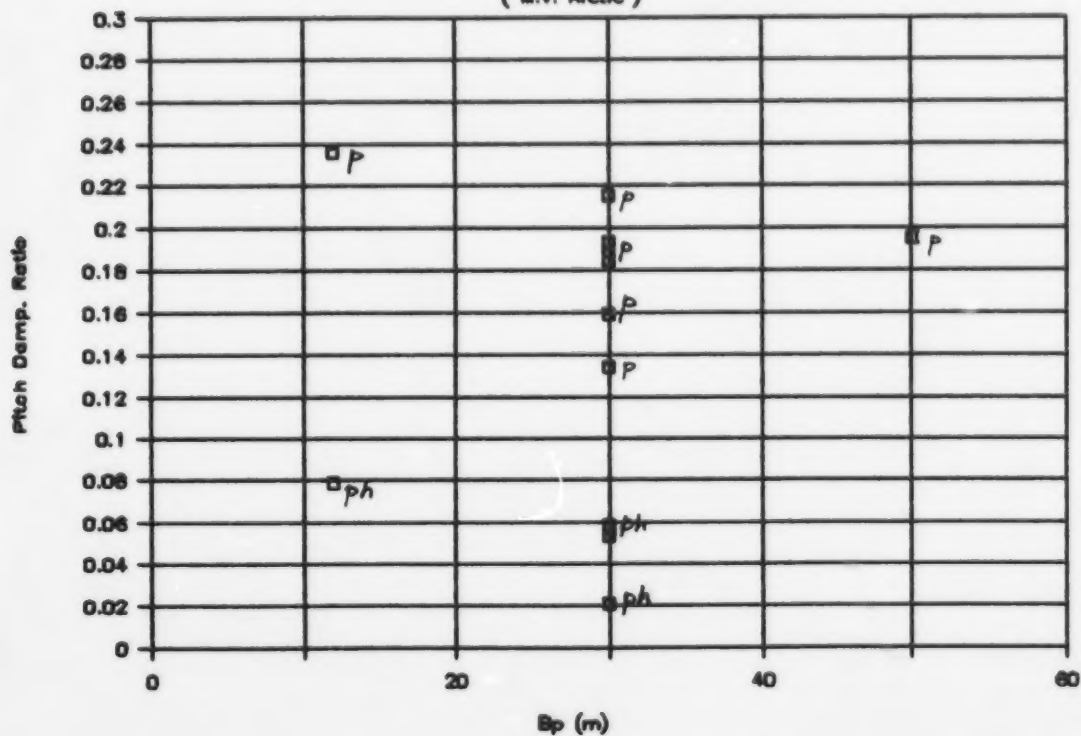
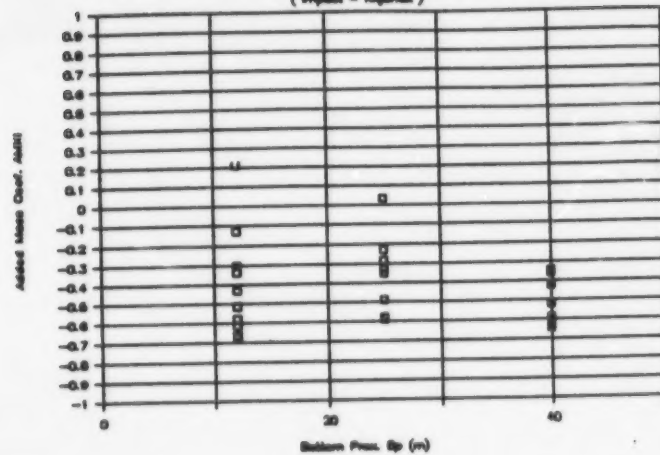


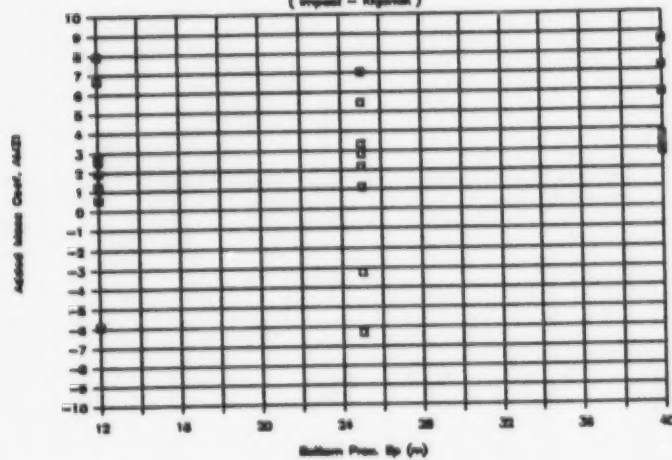
Fig. 52 Damping Ratios During Free Oscillations (M.V. Arctic)



# Bottom Prox. vs Surge AMC ( Impact - Kigoriak )



# Bottom Prox. vs Heave AMC ( Impact - Kigoriak )



# Bottom Prox. vs Pitch AMC ( Impact - Kigoriak )

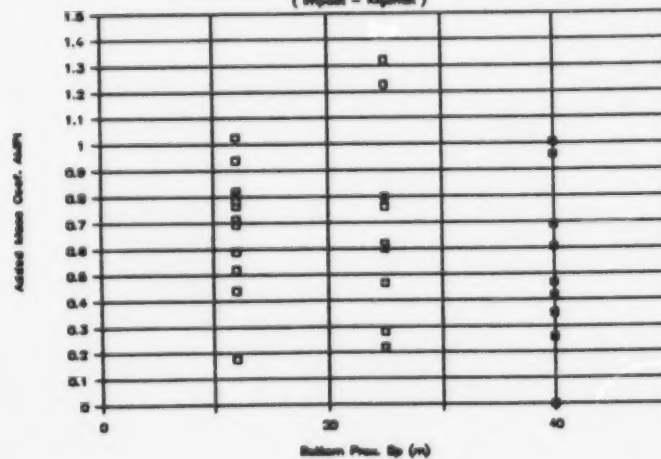
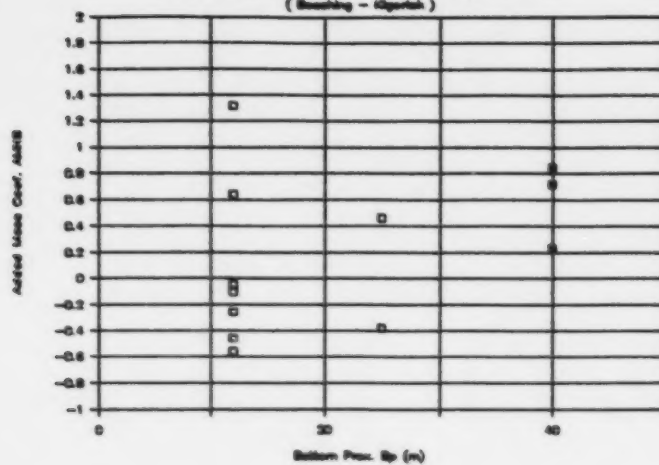


Fig. 53 Impact Added Mass Coefficients (M.V. Kigoriak)



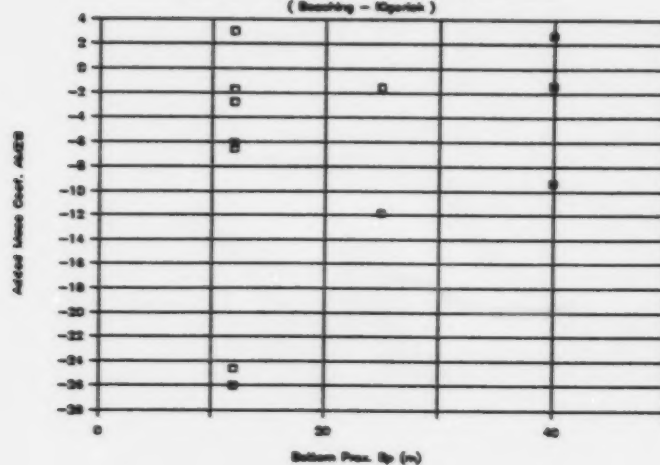
# Bottom Prox. vs Surge AMC

(Beaching - Kigoriak)



# Bottom Prox. vs Heave AMC

(Beaching - Kigoriak)



# Bottom Prox. vs Pitch AMC

(Beaching - Kigoriak)

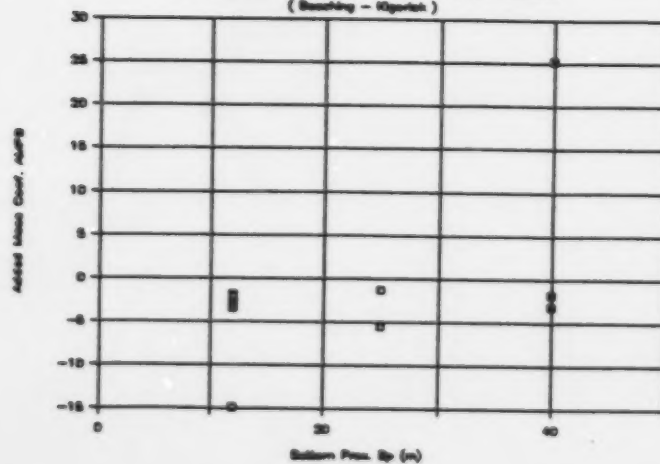
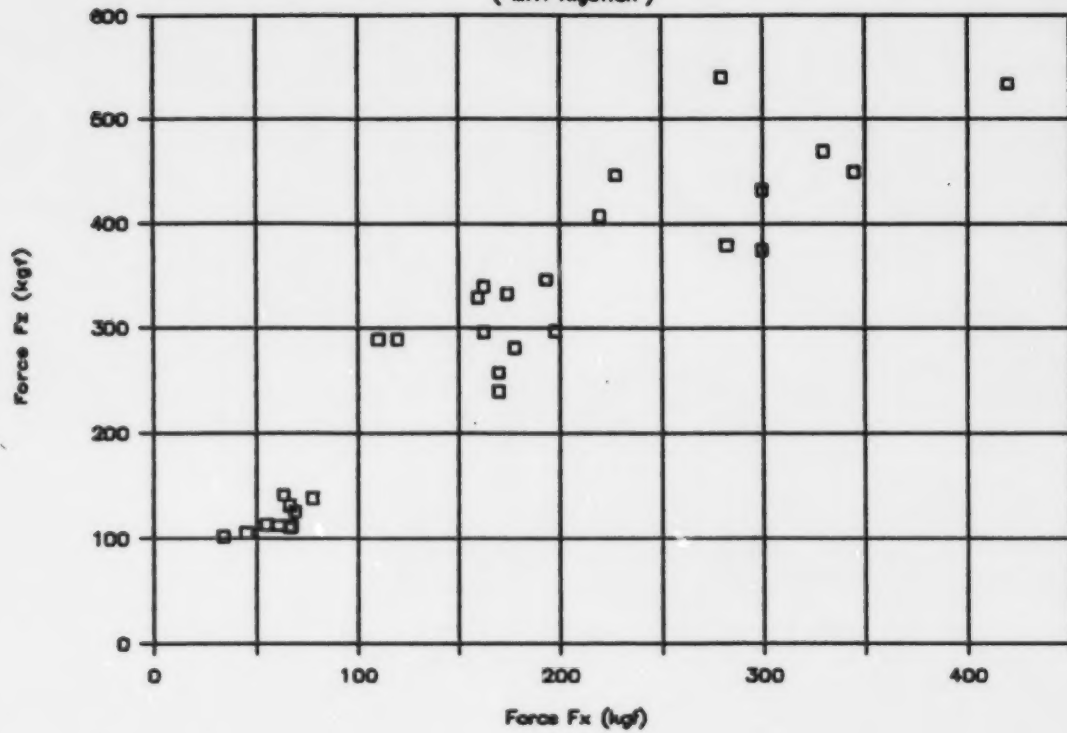


Fig. 54 Beaching Added Mass Coefficients (M.V. Kigoriak)

# **Fx vs. Fz AT IMPACT**

( M.V. Kigoriak )



# **Fx vs. Fz AT Beaching**

( M.V. Kigoriak )

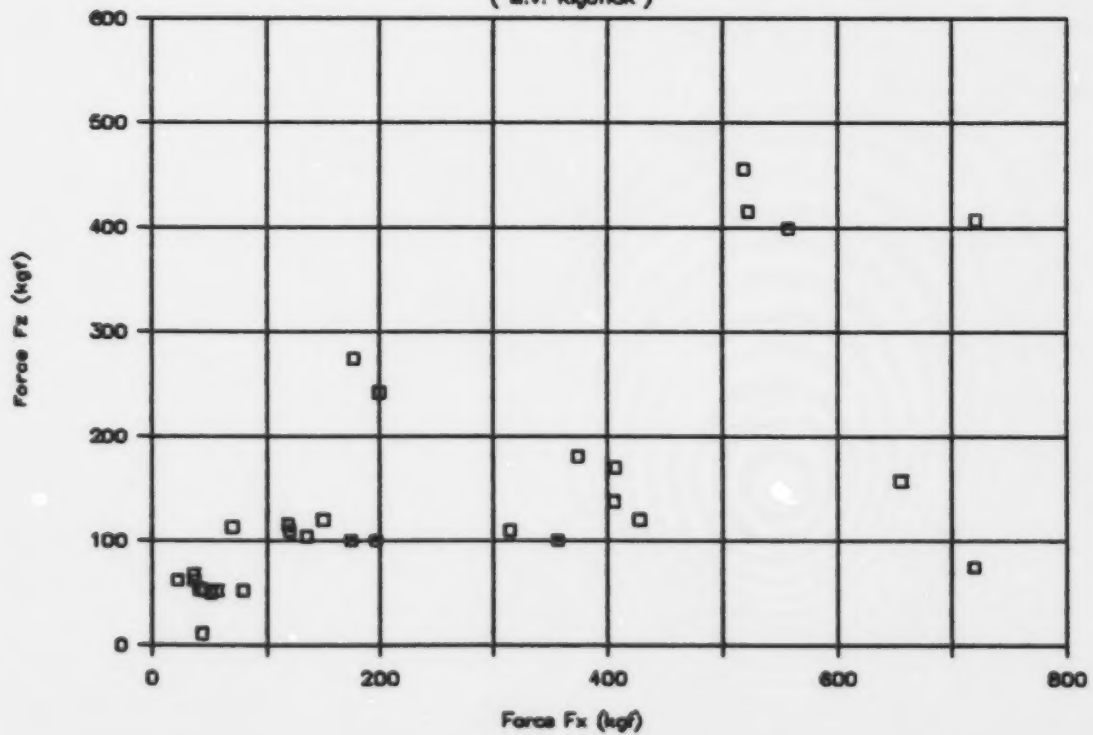
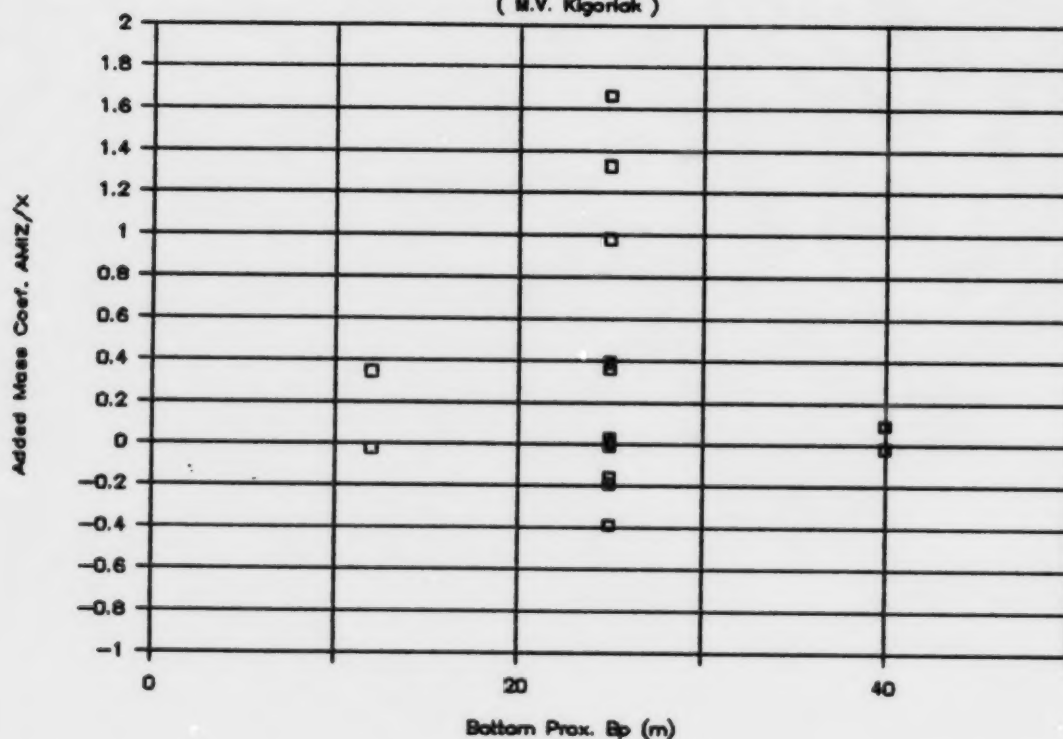


Fig. 55 Relationship Between Vertical and Horizontal Forces (M.V. Kigoriak)

- 122 -  
Bp vs. AMIZ IMPULSE

( M.V. Kigoriak )



Bp vs. AMIP IMPULSE

( M.V. Kigoriak )

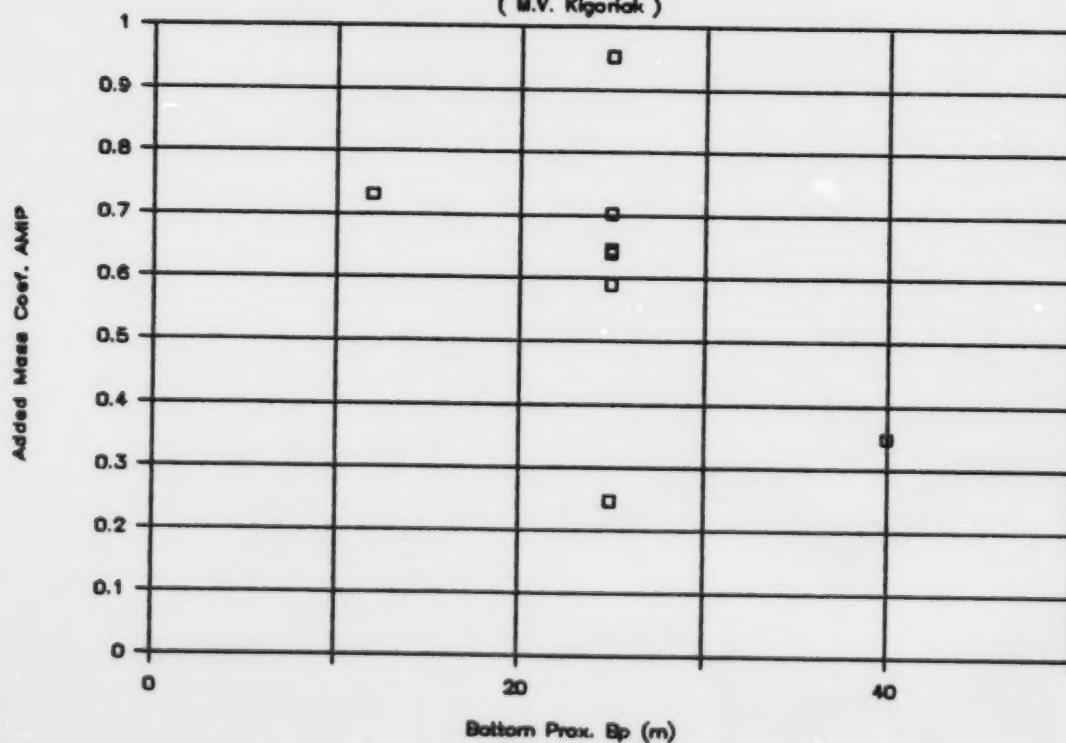
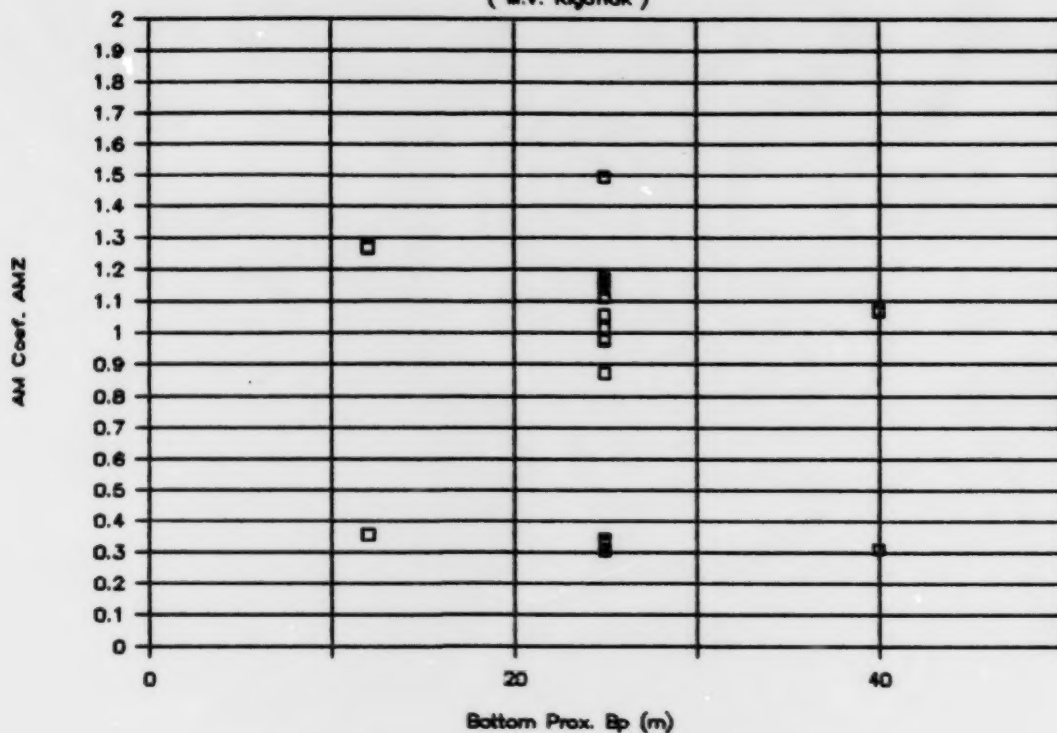


Fig. 56 Added Mass Coefficients at Impulse Peak Force (M.V. Kigoriak)

# Bp vs. AMZ FREE OSC.

( M.V. Kigoriak )



# Bp vs. AMP FREE OSC.

( M.V. Kigoriak )

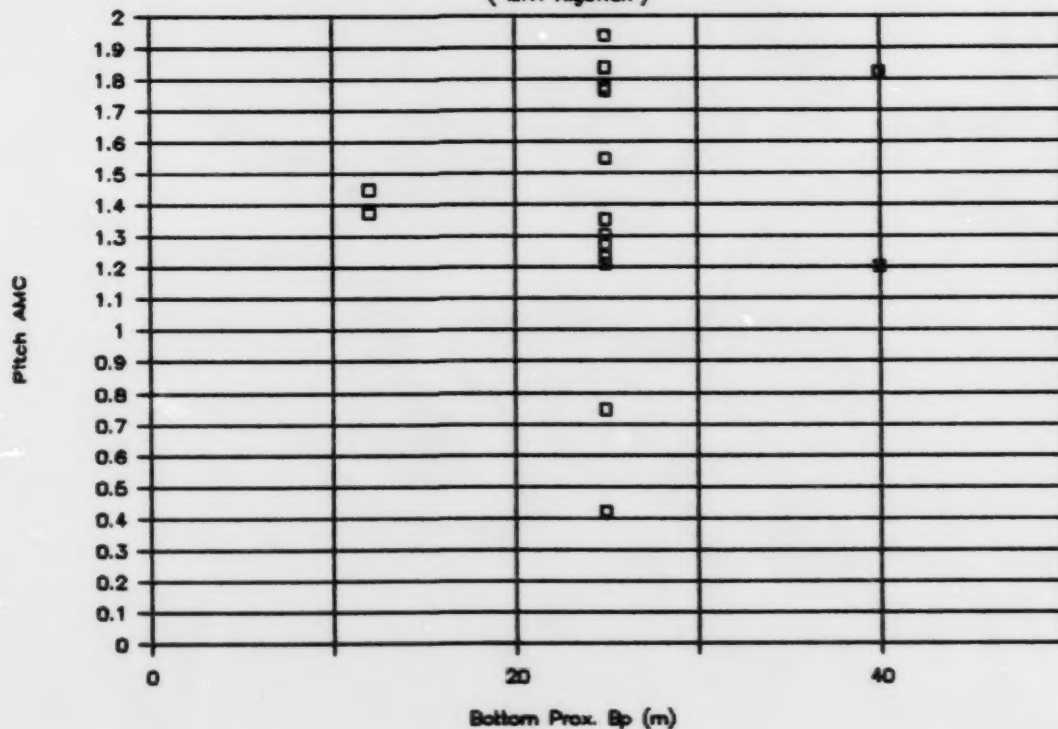
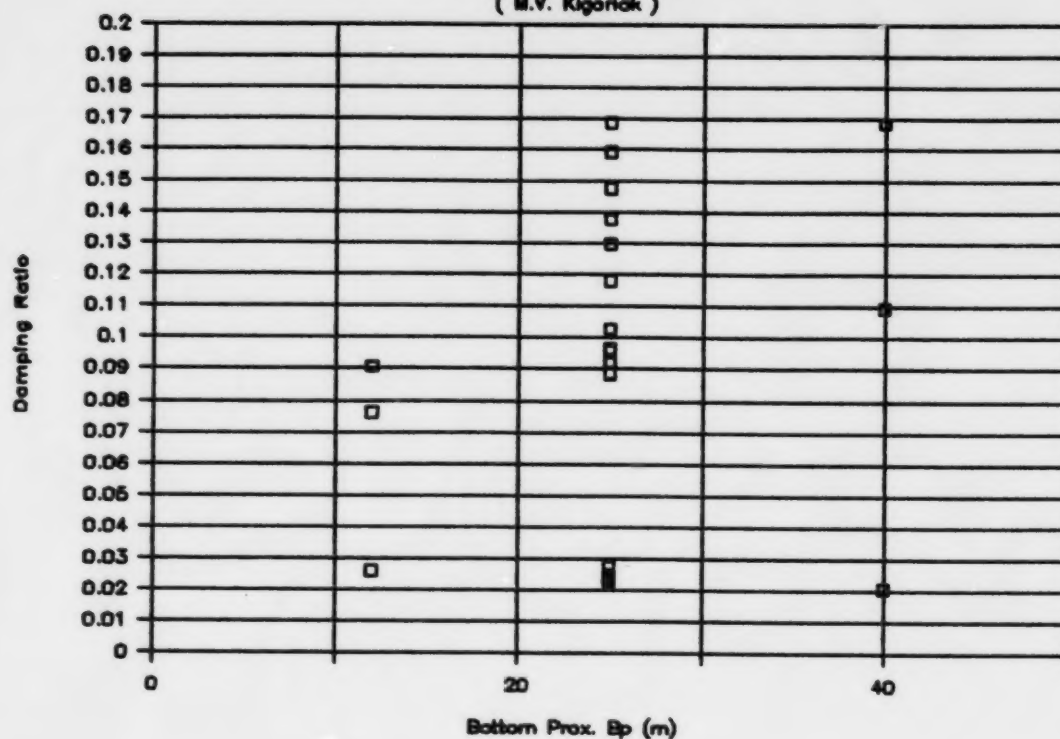


Fig. 57 Added Mass Coefficients During Free Oscillations (M.V. Kigoriak)

# Bp vs. Damping Ratio

( M.V. Kigoriak )



# Bp vs. DAMPP FREE OSC.

( M.V. Kigoriak )

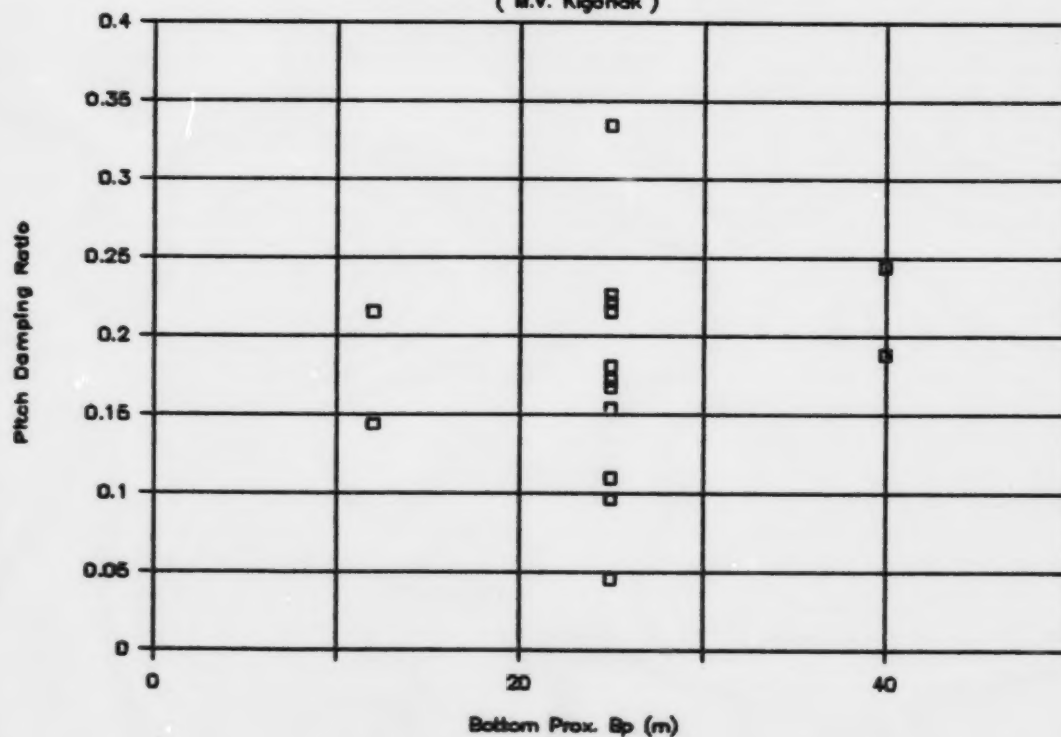


Fig. 58 Damping Ratios During Free Oscillations (M.V. Kigoriak)

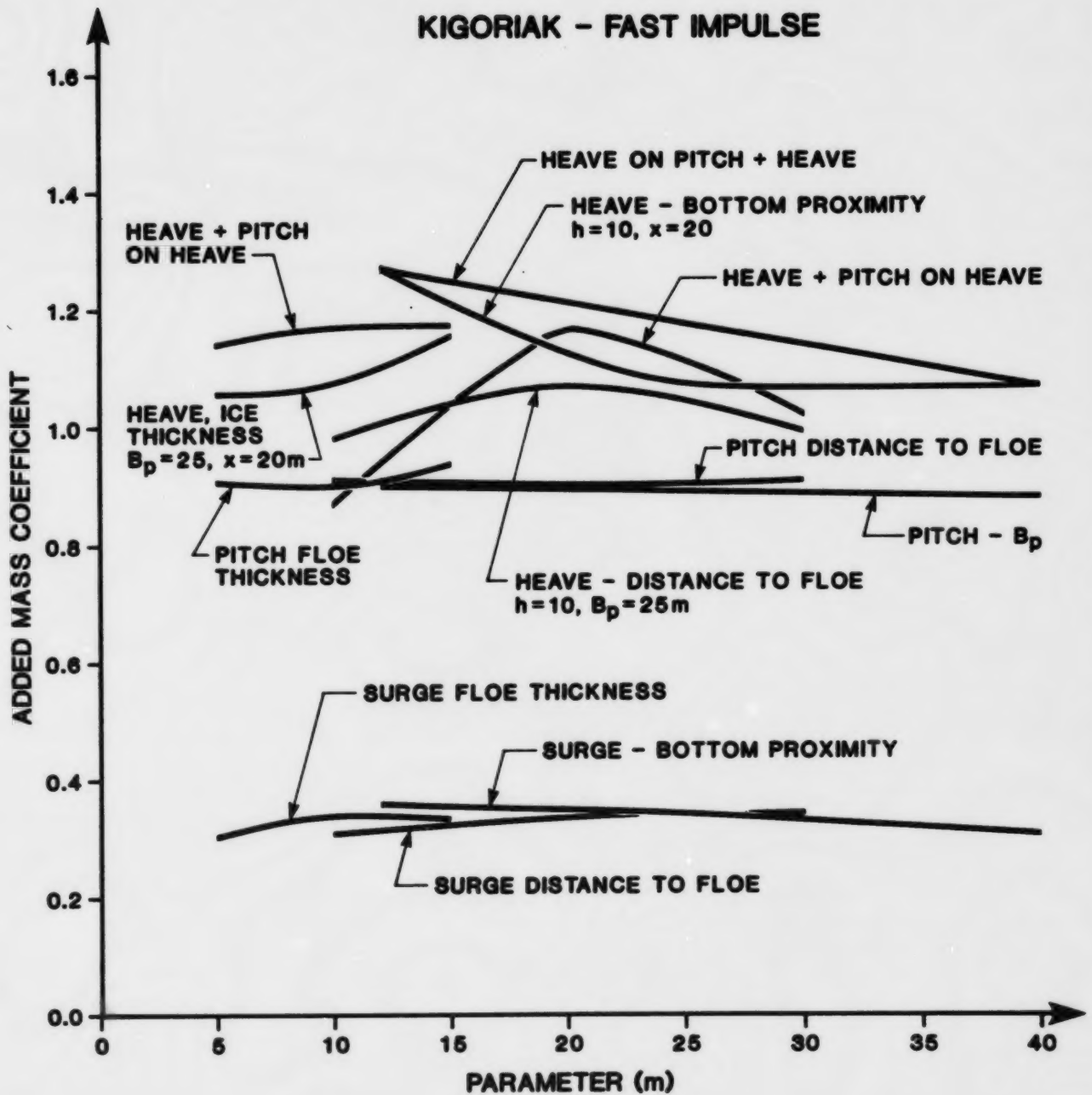


Fig. 59 Added Mass Dependence on Bottom Proximity, Ice Thickness, and Distance to Ice



### 3.5 Finite Element Analysis

As stated in Section 2.5 a skeletal structural framework was developed to model the structural response of the Kigoriak hull. The results for the first few frequencies of interest for this skeletal model vibrating in air are compared in Table 13 with results obtained from a far more sophisticated three dimensional representation of the vessel (Ref. 35). It should be borne in mind that the objective of the exercise was to develop a simplistic three dimensional representation of the dynamic properties of a vessel having the hull form of the Kigoriak. Thus the results indicated in Table 13 are seen to be in reasonable agreement as to satisfy this requirement. Since a set of frequency results were available for an open water condition from Ref. (35) and given the budget constraint which enabled only one eigenvalue analysis to be performed with fluid coupling, it was decided to run the skeletal model in the presence of an ice floe 4.9 m thick. The natural frequencies for both wet and dry modes of the skeletal structure in the presence of an ice edge are shown in Table 14 together with computed values of added mass coefficient derived as indicated in Section 2.5.1.2.

TABLE 13  
Comparison of Natural Frequencies (Hz) for the Skeletal  
and Full 3D Idealisation of Kigoriak

| <u>Mode No.</u> |               | <u>Added Mass</u><br><u>Skeletal</u> | <u>Added Mass</u><br><u>3D Model Ref. (35)</u> |
|-----------------|---------------|--------------------------------------|------------------------------------------------|
| 1               | Surge         | 0.026                                | 0.04                                           |
| 2               | Heave & Pitch | 0.212                                | 0.214                                          |
| 3               | Pitch         | 0.26                                 | 0.271                                          |
| 4               | 2 node        | 4.31                                 | 4.01                                           |
| 5               | 3 node        | 9.499                                | 8.39                                           |
| 6               | 4 node        | 15.049                               | 13.36                                          |



TABLE 14  
Comparison of Natural Frequencies (Hz) for Skeletal  
Idealisation of Kigoriak In Air and Adjacent to a 4.9 m Ice Edge

| Mode No. | In Air (Hz) | In Water* (Hz) | Added<br>Mass | Remarks       |
|----------|-------------|----------------|---------------|---------------|
|          |             |                | $\lambda$     |               |
| 1        | 0.026       | 0.025          | 0.082         | Surge         |
| 2        | 0.212       | 0.144          | 1.167         | Heave & Pitch |
| 3        | 0.26        | 0.179          | 1.11          | Pitch         |
| 4        | 4.31        | 3.253          | 0.76          | 2 node        |
| 5        | 9.499       | 7.551          | 0.58          | 3 node        |
| 6        | 15.049      | 12.4           | 0.47          | 4 node        |

\* with an ice edge 4.9 m thick at bow.

Figures 60 through 65 illustrate both the modal shapes and the distribution of hydrodynamic force in the longitudinal and athwartships direction for the first six modes of the skeletal model of the Kigoriak hull, vibrating in air and in water in close proximity to an ice edge. The first three modes are the rigid body responses of the vessel in surge, heave and pitch respectively.

It is clear from Figures 60a and 61a that the surge and heave rigid body wet modes are not quite pure translations. The dotted line shown in each of these figures depicts the corresponding dry mode shape and the clear inference is that the hydrodynamic effects on the curved body from results in coupling between the rigid body modes. It is interesting to examine both the distribution and sign of the hydrodynamic forces acting on the vessel for each of the modes shown. As can be seen these forces are discretised at the node points along the hull girder and represent the integration of the hydrodynamic pressures acting locally on the projected areas of the hull in the longitudinal (X) and vertical (Z) directions. The forces as will be seen later have been calculated assuming an arbitrarily chosen maximum displacement of 1 m in each mode shape.

Examining Figure 60a we can see that the wet surge mode in addition to having a unit (1 m) displacement in the X direction also experiences pitching motion. It will be recalled that an essential assumption in the development of the finite element idealisation was a requirement for no flow separation to occur at the hull surface. Thus if the vessel surges in the positive X direction one can imagine a positive pressure build up ahead of the vessel while at the aft end fluid will be "sucked in" to fill the void. Figure 60b illustrates the forces acting on the vessel so that the effect of positive pressure at the forward end is to oppose motion which the suction at the aft end will similarly attempt to inhibit forward movement. Thus the forces in the longitudinal direction both act in a negative sense. The magnitude of the forces also reflect the diminishing areas on which they act as the ends of the vessel are reached. The effect of the parallel mid body on the forces that is no change in projected area in the longitudinal direction is quite evident in Figure 60b.

By similar reasoning the pitching effect, for the position shown in Figure 60a, will further increase the positive pressure forward, by attempting to "all press" the fluid while the upward motion of the aft end will attempt to "draw" the fluid up with it. Figure 60c illustrates how the hydrodynamic force in the vertical direction changes along the length of the vessel. Note that at the point of zero pitching, approximately 46 m from the aft end, the force undergoes a change of sign. Again the variation in the magnitude of the discretised forces reflects the changing projected areas on which the pressure component can act.

Figure 61c has the same basic shape as Figure 60c since they both represent the effect of pitching motion on the vertical hydrodynamic force. The magnitudes of the forces are higher in the pure pitching mode case because higher values of accelerations have been used. (As will be seen later the accelerations in a particular mode have been scaled to the square of the frequency). The values of  $\pm 10$  m for the vertical displacement of the vessel in the pitch mode are simply to facilitate display, they have no absolute meaning. The hydrodynamic force in the longitudinal direction shown in Figure 61b is a coupling effect. As described previously, suction pressure will be induced aft of the pitch node point while the fluid will experience a positive pressure rise forward. The integrated effect of these pressures over the local

projected area in the longitudinal direction will produce forces acting the negative global X direction.

The combined "heave and pitch" mode of Figure 62 is interesting. It probably would be more correct to refer to the mode as an impure heave mode since there is no nodal point in the mode shape. One can, however visualise a superimposed pitching action on top of a steady heave component, hence the terminology used. The positive, upward movement along the entire hull causes negative pressure. Thus Figure 62c illustrates negative hydrodynamic forces acting along the entire hull. As noted previously suction pressure at the aft end results in longitudinal forces acting in the negative X direction. In this mode suction pressure in the forward region will attempt to "pull" the vessel forward hence the positive values for longitudinal force shown in Figure 62b.

Figures 63, 64 and 65 are concerned with the first three vertical flexural mode shapes. Following a similar reasoning as before that is, an upward movement of the hull induces negative (suction) pressure while downward movement results in a positive pressure field, the distribution and signs of the vertical hydrodynamic forces in Figures 63, 64 and 65c are consistent with the associated mode shapes shown in Figures 63, 64 and 65a. Similarly having determined whether the local pressure regime is positive or negative the direction of the global longitudinal force at each node point or the hull can be established for each of the mode shapes. These distributions have been plotted in Figures 63, 64 and 65b respectively.

If the vessel were to be given uniform rigid translational accelerations in each of the three orthogonal directions in turn there would be nine ensuing hydrodynamic forces produced on it as a consequence of fluid motion. The SESAM program produces these nine force components as a by product of the eigenvalue analysis. The forces are given below in Table 15 for both the skeletal model in the presence of an ice edge and compared with those obtained from the 3-dimensional open water fluid idealisation adopted in Ref. (35). It is important to note that although the skeletal structural model appears pictorially to be transparent to the fluid in actual fact the fluid is attached to the framework at the nodes below the waterline while the application of

the boundary condition of zero flow perpendicular to the surface ensures that whatever the nature of the structural representation the fluid "sees" a solid body.

**TABLE 15**  
**Comparison of Hydrodynamic Forces on the Kigoriak Hull due to**  
**Pure Unit Translational Acceleration in the Orthogonal Directions X, Y and Z**

|   | <u>Previous Model Ref.(35)</u><br>(Open Water) |          |          | <u>Skeletal Model</u><br>(With Ice Edge) |          |          |
|---|------------------------------------------------|----------|----------|------------------------------------------|----------|----------|
|   | <u>X</u>                                       | <u>Y</u> | <u>Z</u> | <u>X</u>                                 | <u>Y</u> | <u>Z</u> |
| X | - 0.153                                        | 0        | - 0.079  | - 0.163                                  | 0.002    | 0.0185   |
| Y | 0                                              | - 1.658  | 0        | 0.002                                    | - 1.867  | 0        |
| Z | - 0.0185                                       | 0        | - 4.893  | - 0.079                                  | 0        | - 4.44   |

The forces in the above table are in MegaNewtons, as expected the confining effect of the ice edge serves to increase the hydrodynamic force in all three rigid body modes. The increase is in the range 6 - 10%. Many more fluid elements were used with the present model so it should provide a correct representation of the flow region.

Since the forces were produced by unit acceleration then the associated added mass values are numerically the same. Thus the hydrodynamic added mass in pure heave is  $8.88 \times 10^6$  Kilograms (doubled because only half the hull is modelled). The actual displacement of the vessel during these analyses was taken to be 6806 tonnes thus indicating an added mass coefficient of 1.3 in pure heave. Similarly the use of the hydrodynamic force of  $2 \times 0.153$  MN in the X (surge) direction leads to an added mass coefficient of 0.0899 in surge.

Returning to the discussion of Figures 60 to 65 it is interesting to observe the relatively small differences in the flexural mode shapes in the wet and dry conditions. It is important to point out that the scale of the displacement axes for each mode



shape has been distorted to facilitate display and derives from simply setting the largest relative amplitude coefficient in each mode shape to 10 m. Even more importantly the magnitude of the hydrodynamic forces computed in accordance with equation (50), see Sec. 2.5.1.2 on the basis of a maximum displacement of 1 m and shown in Figures 60 to 65 are unrealistically high. These forces do of course require to be scaled by a factor  $a/(2\pi f)^2$  if they are to have real meaning in design where  $a$  is the amplitude. For example if the maximum acceleration in the first flexural mode is not to exceed  $a = 0.5$  g anywhere on the hull then the scaling factor has the value,

$$C_s = \frac{0.5 \times 9.81}{(2\pi \times 3.25)^2} = 0.0117628$$

and from Figure 62 the resulting maximum hydrodynamic force due to the added mass effect has a value of 0.58 MN. A similar exercise for the 3 node vertical results in a maximum hydrodynamically induced force of 0.407 MN: the force reduction in the higher mode being a reasonable expectation.

The objective in plotting discretised hydrodynamic forces at each node for the several modes was so as to enable direct comparison to be made between the results from various ice edge conditions. However as has been pointed out earlier although the methodology and fluid element meshes had been derived to model bottom proximity and floe thickness effects the associated computing costs were far in excess of the resources available. Figures 60 to 65 nevertheless show that hydrodynamic pressure forces can be calculated from the developed methodology.

The question might quite legitimately be asked, "why not plot added mass distributions along the hull rather than force?" To answer this, one must consider the form of Equation 50. The hydrodynamic mass matrix is fully populated though certainly diagonally dominant. The presence of off diagonal terms is due to the important effect of coupling through the fluid. For instance if only one point on the hull experiences a unit acceleration then one could expect a high hydrodynamic pressure at that point while nodes in the neighbourhood would see much lower pressures, the effect eventually diminishing with distance. The hydrodynamic mass matrix is thus a tabulation of pressure influence coefficients, with the elements of

each row representing the back pressure effect at that node point of a unit perturbation at a given node. Thus as each node point vibrates simultaneously hydrodynamic pressure forces are set up at all other nodes. The net force at any node is thus the summation of all the pressure effects being generated around it. This summation is performed by the matrix multiplication indicated in Equation (50). It would be incorrect therefore to take these individual nodal forces and simply divide them by their individual nodal accelerations to yield a point added mass since this would negate any coupling and simply result in a diagonal mass matrix which is not truly representative of the fluid effect.

While it has not proved possible to fully examine the effects of edge proximity on hydrodynamic flow for the three dimensional case, some additional tests were performed using a two dimensional idealisation. A rectangular ship crosssection is shown in Figure 16 and interest is confined to the pure heave case. By varying the water depth the effect of the hydrodynamic pressure on the bottom and vertical sides of the box was readily calculated and the results are shown in Figure 17. These pressures were integrated to provide the hydrodynamic force in the vertical direction. Since this force was produced by prescribing unit vertical acceleration on the box it has the same numerical value as the added mass. Thus it is clear from Figure 17 that bottom proximity increases the hydrodynamic pressure and hence added mass. The effect of restricting the width of the water way was also investigated by halving the distance from the edge of the box to the fluid boundary while using the largest water depth. This has quite a pronounced effect on the hydrodynamic pressures acting on the box since effectively the radiation potential is reflected from the boundary rather than diminishing to zero.

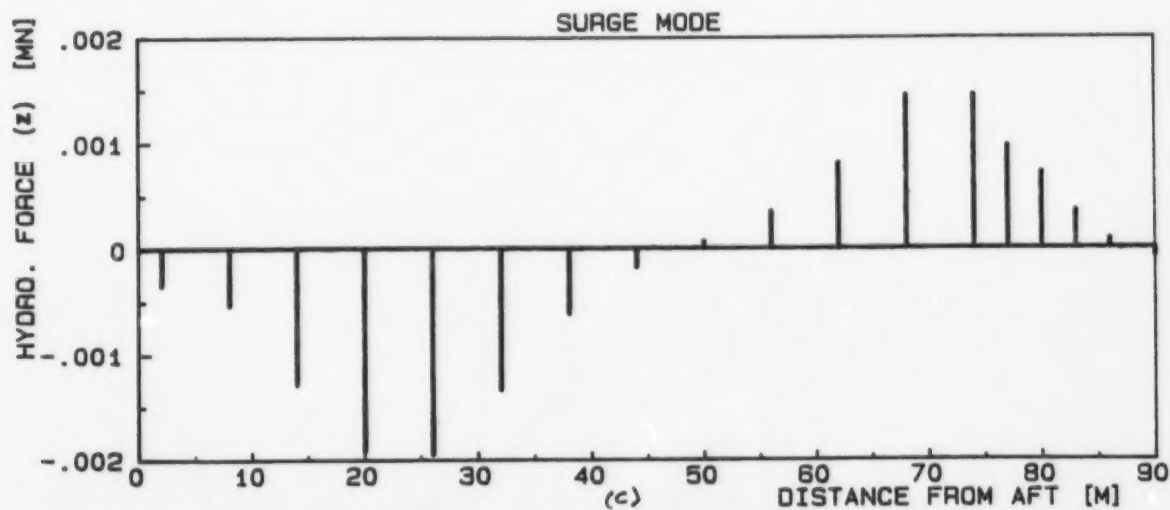
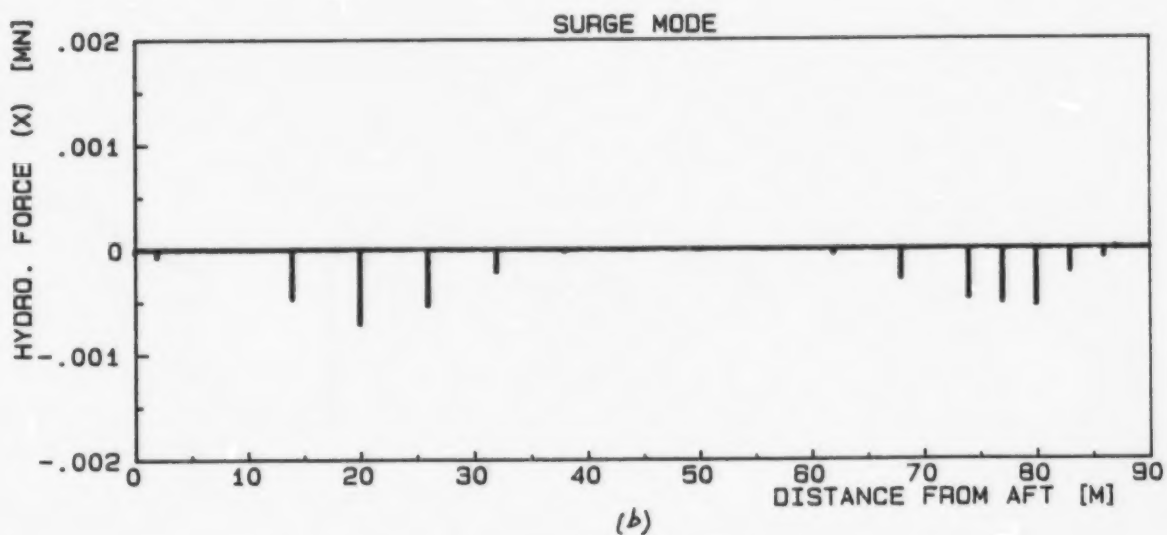
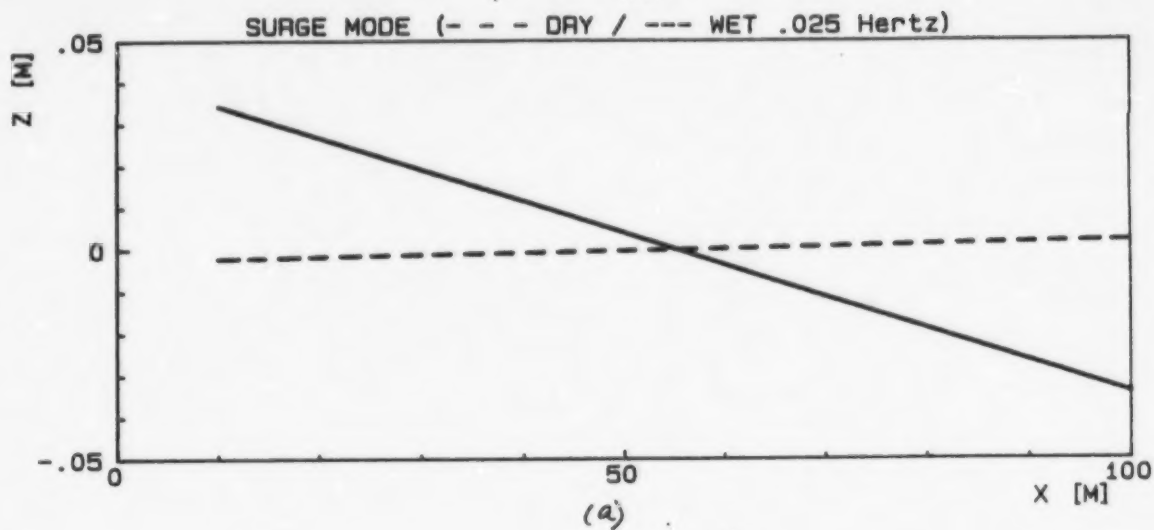


Fig. 60 Wet and Dry Surge Mode Shapes and Distribution of Hydrodynamic Force Open Water from F.E.M. Modelling of Kigoriak



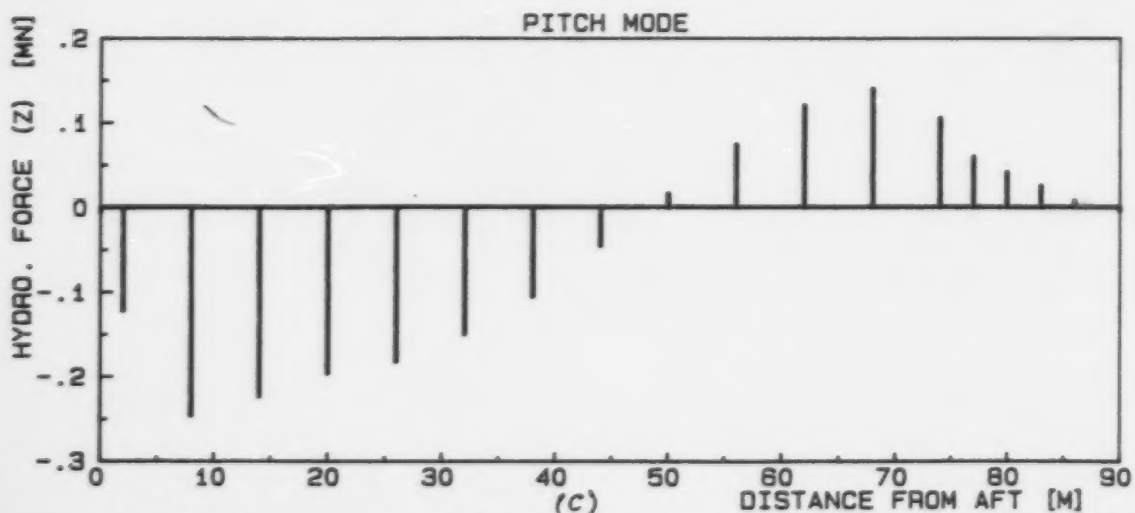
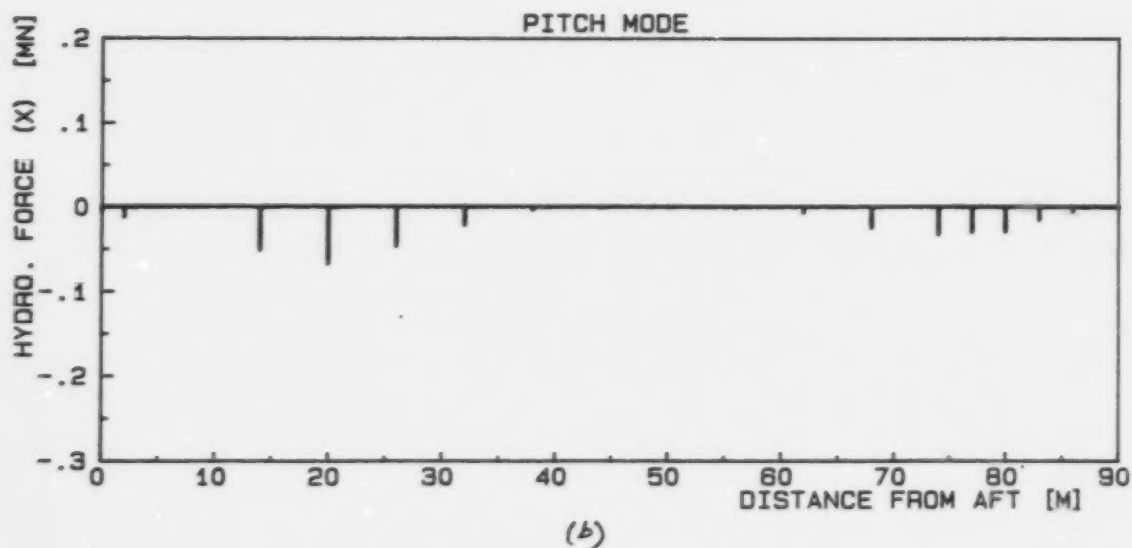
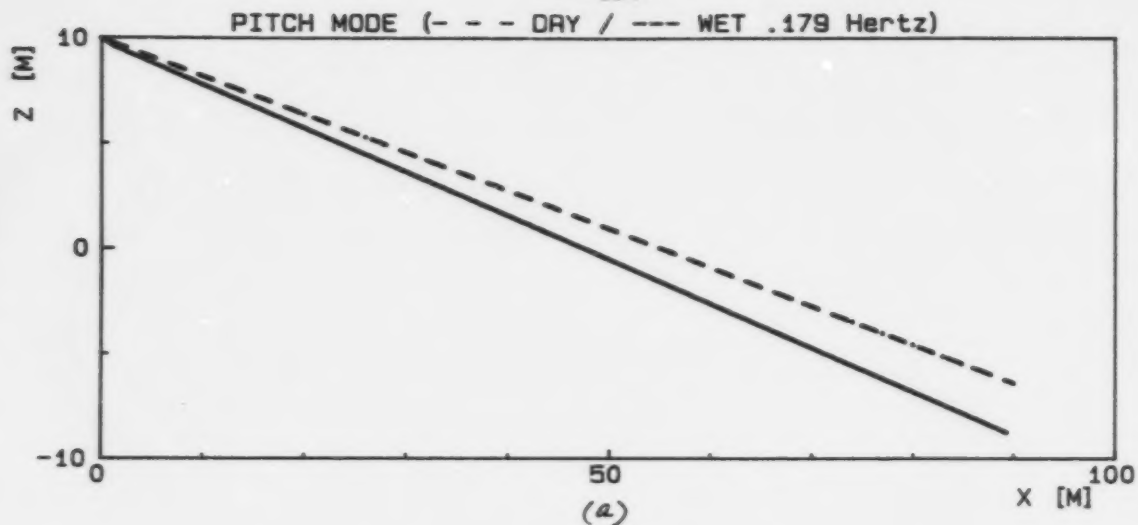


Fig. 61 Wet and Dry Heave and Pitch Mode Shapes and Distribution of Hydrodynamic Force Open Water from F.E.M. Modelling of Kigoriak

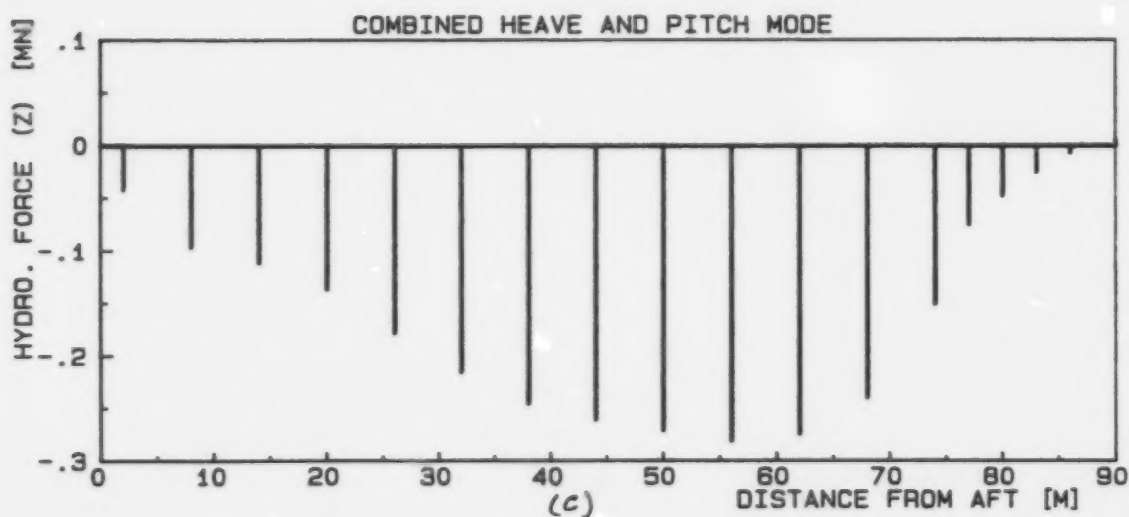
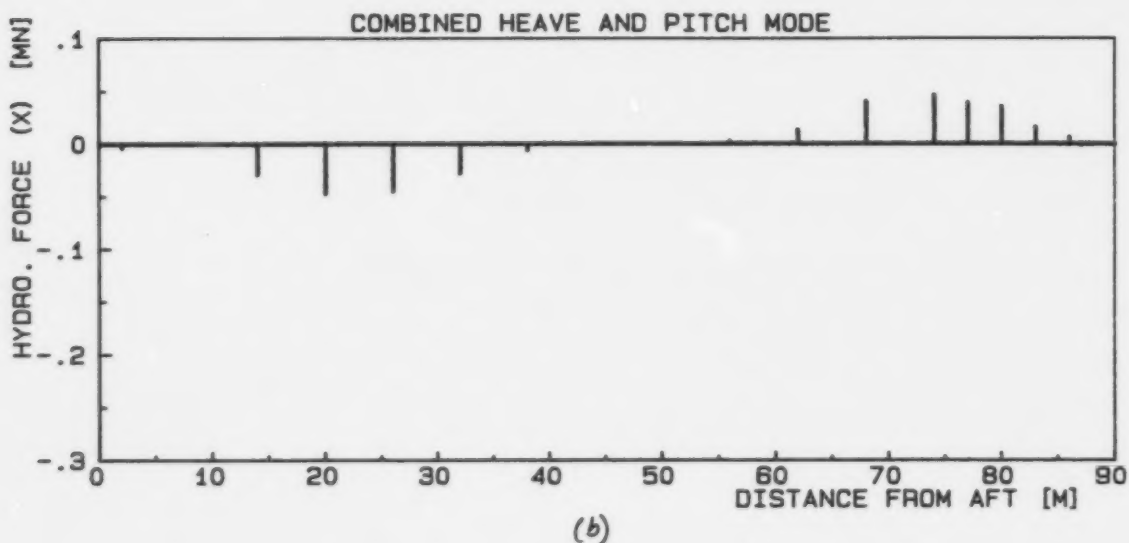
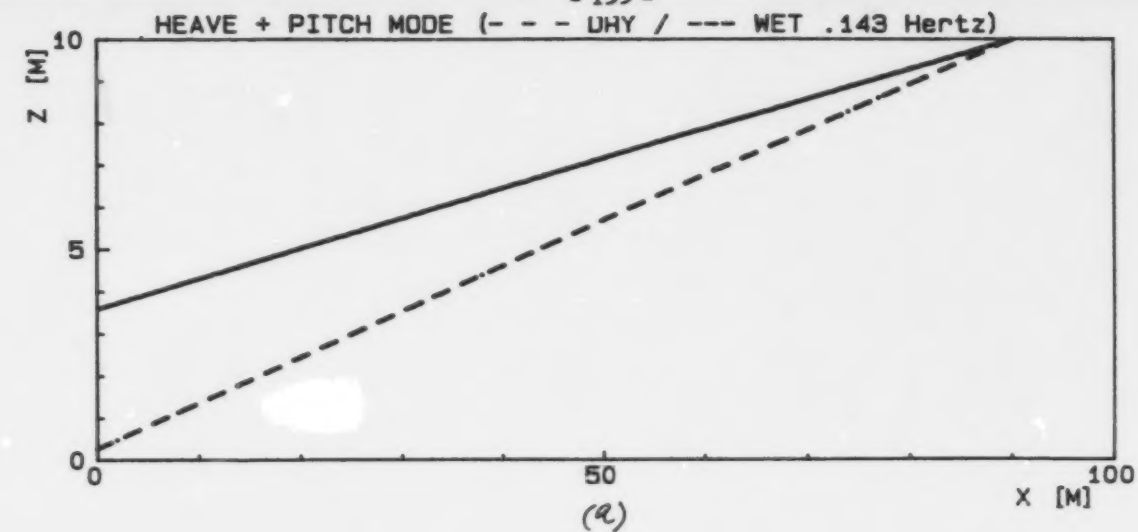


Fig. 62 Wet and Dry Pitch Mode Shapes and Distribution of Hydrodynamic Force Open Water from F.E.M. Modelling of Kigoriak

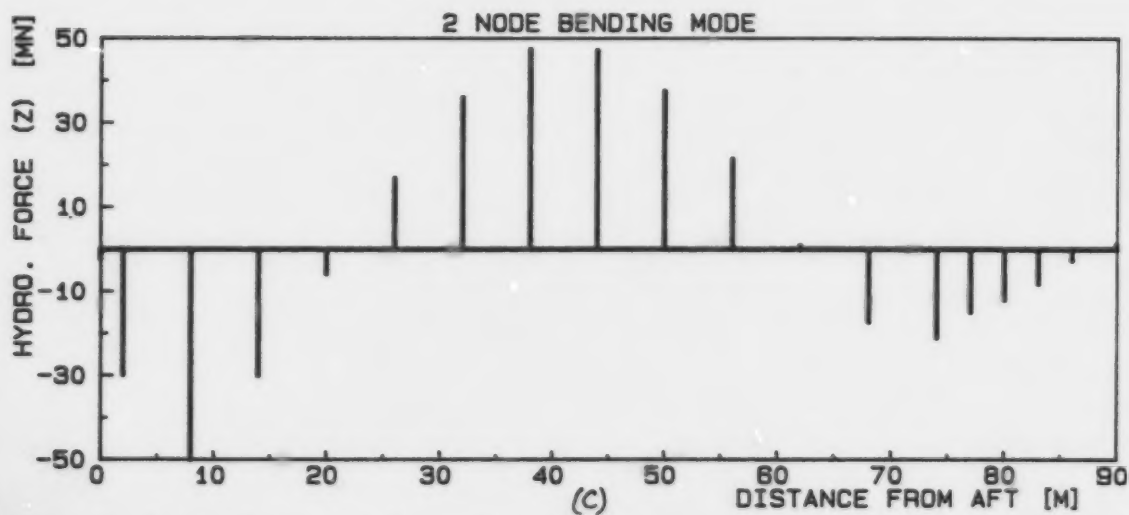
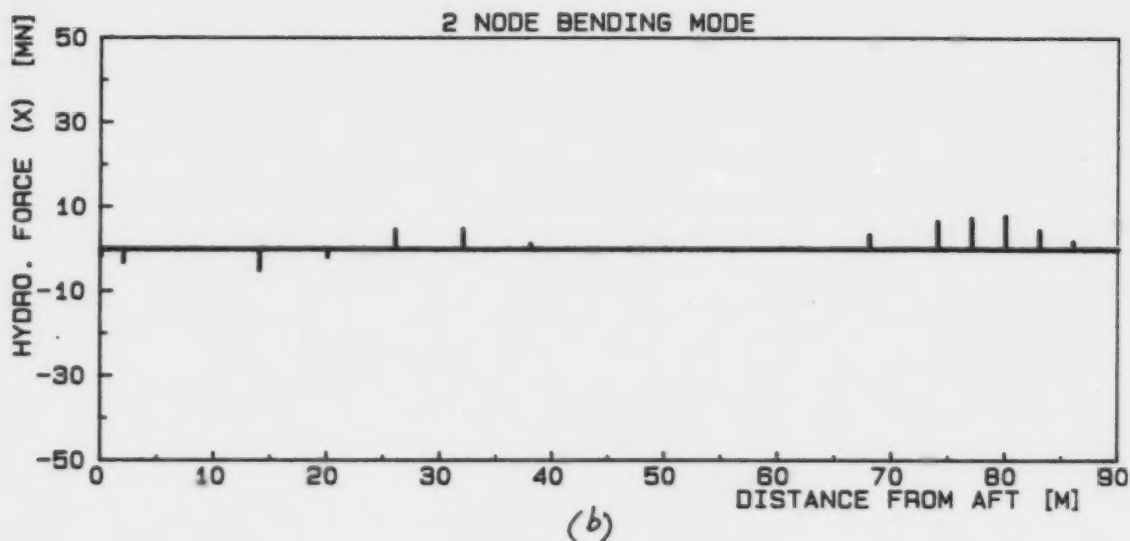
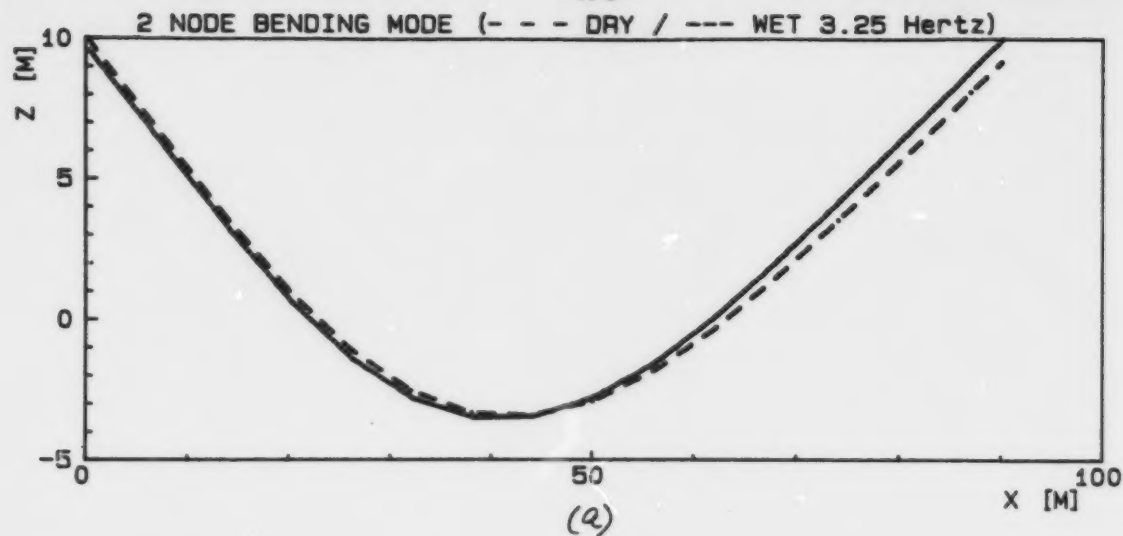


Fig. 63 Wet and Dry 2 Node Bending Mode Shapes and Distribution of Hydrodynamic Force Open Water from F.E.M. Modelling of Kigoriak

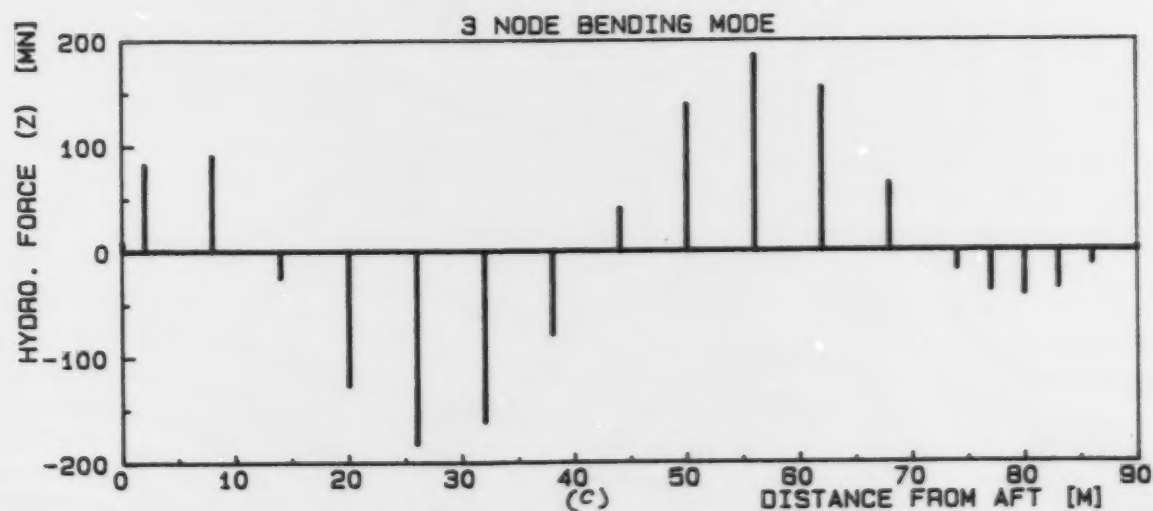
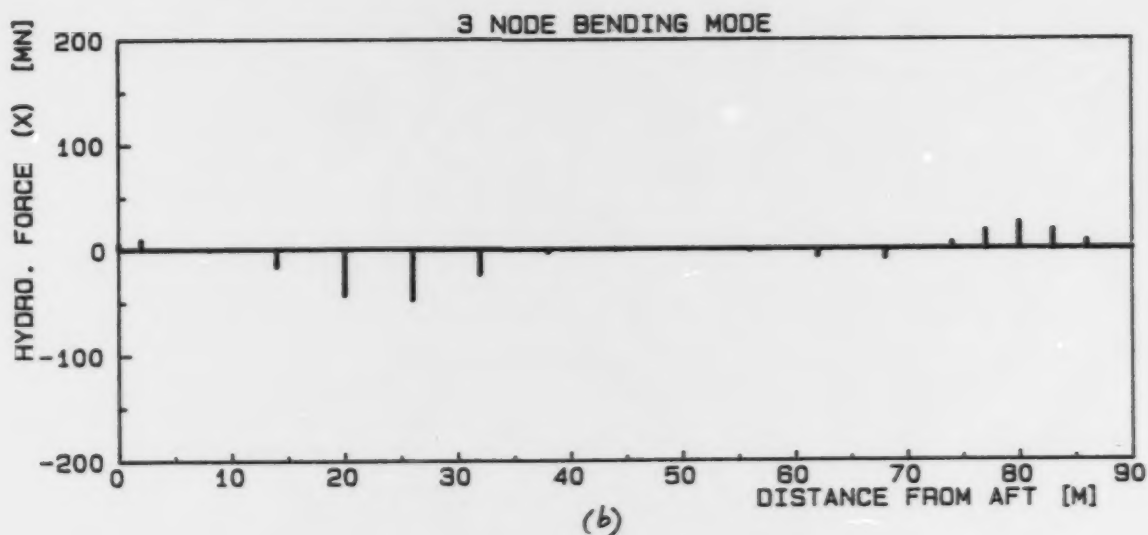
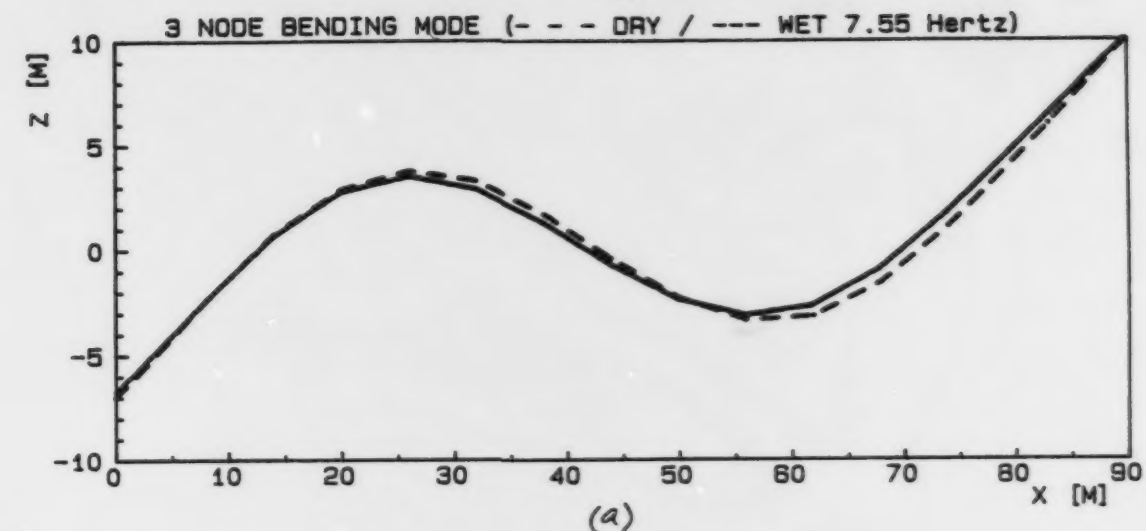


Fig. 64 Wet and Dry 3 Node Bending Mode Shapes and Distribution of Hydrodynamic Force Open Water from F.E.M. Modelling of Kigoriak

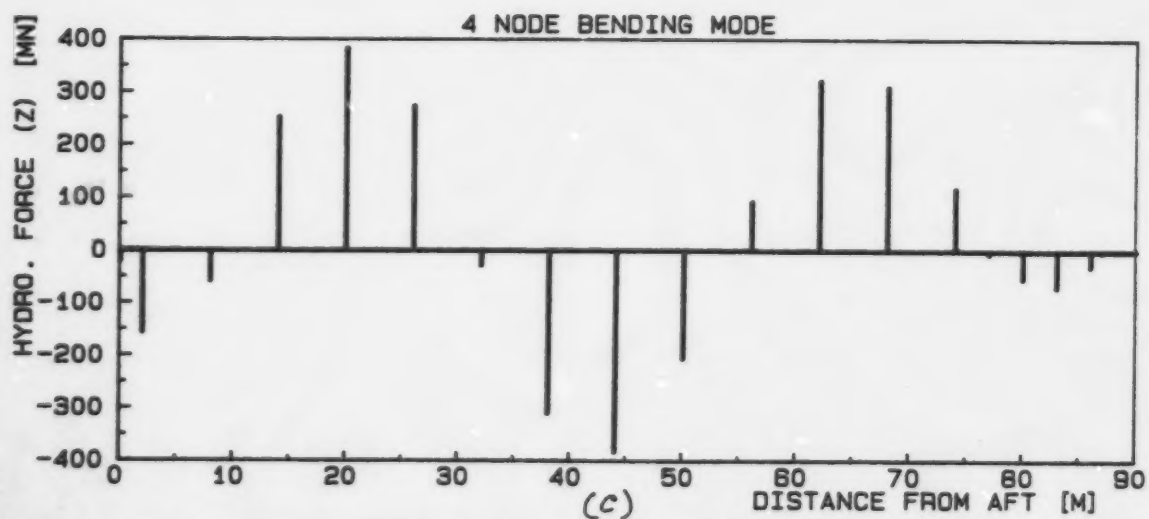
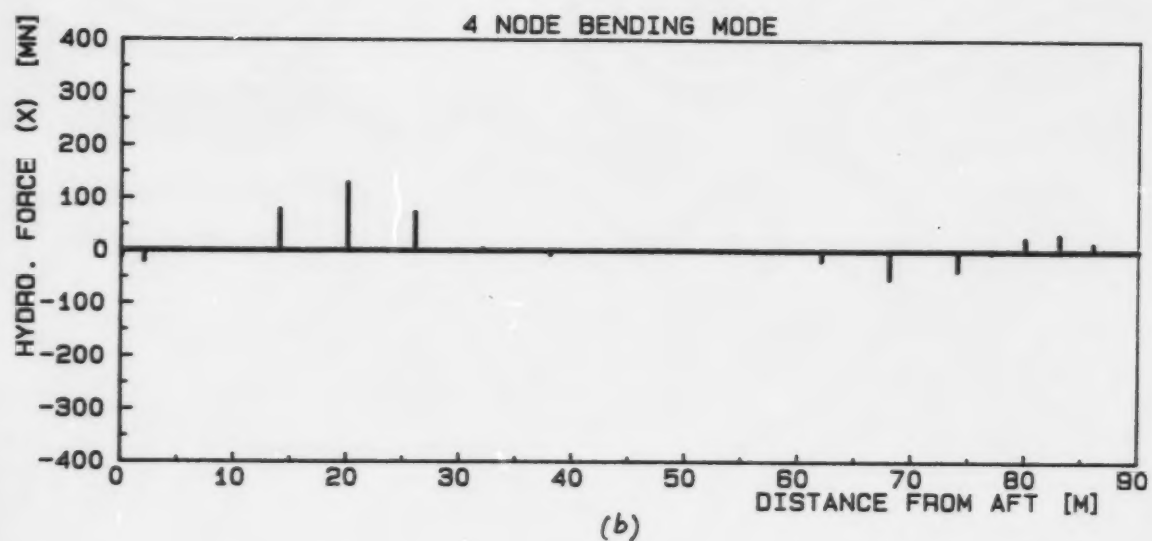
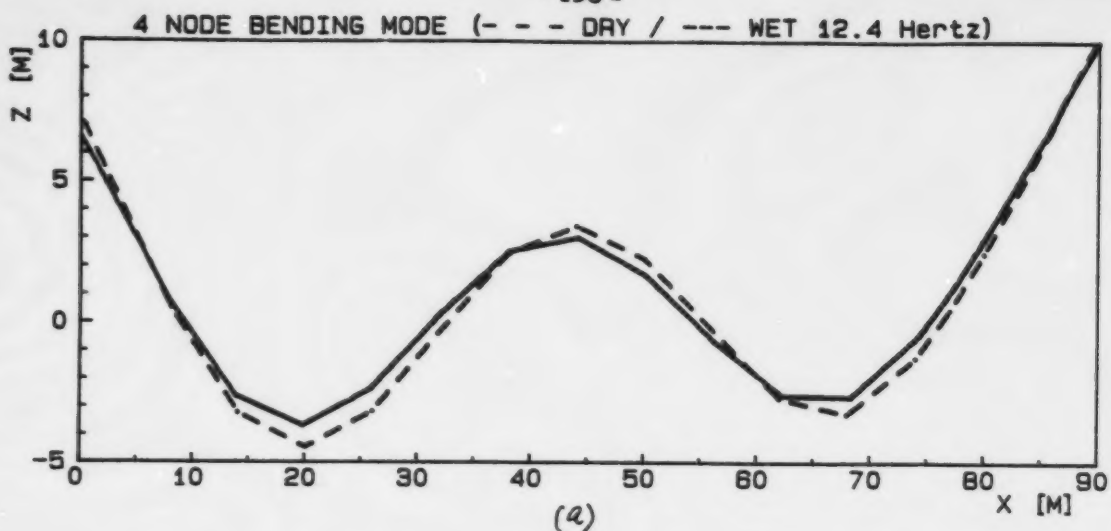


Fig. 65 Wet and Dry 4 Node Bending Mode Shapes and Distribution of Hydrodynamic Force Open Water from F.E.M. Modelling of Kigoriak

### 3.6 Comparison of Results

The results from the analytical model development, the Kigoriak full scale data analysis, the model tests, and the Kigoriak finite element modelling have been presented in Sections 3.2 to 3.5. The comparison between these results is not straight forward due to the format differences in which each model presents the output results. However, it will suffice herein to compare the added mass values of the vessel in the rigid body modes prior to impact and at impact.

The analytical model (Secs. 2.2 and 3.2) predicts added mass coefficients of 0.05 for surge and 1.1 for heave and pitch. Coupling coefficients were predicted to be insignificant except for the pitch-on-heave value of - .013. The analytical model assumes these values to be applicable for both the approach and impact phases.

The full scale data gave vertical added mass coefficients at impact of 1.04 (average of the six rams presented in Fig. 38 - 43). This vertical coefficient is the superimposed effect of heave, pitch and flexible modes of vibration. The surge added mass coefficients vary between - 0.1 to 0.5 for a set of 16 rams analysed using peak impact forces and accelerations. The average value of 0.2 is reasonable.

The model tests did yield reasonable surge or heave added mass coefficients for the impact phase. The pitch added mass coefficient of 0.5 - 1.325 entails most of the data points. No reasonable added mass coefficients were possible to calculate during the beaching phase due to the small accelerations measured during beaching and the presence of a vibratory component in the acceleration signal. The impulse tests gave more reasonable values for the added mass coefficients. The maximum surge added mass coefficient measured was 0.1. The heave added mass coefficient of 1.66 was measured due to an impulse applied in the bow. The max pitch added mass coefficients measured was 0.95. The values were estimated at the time of the peak force during the impulse. The oscillation tests from Impulse tests however gave much more reasonable values for the added mass coefficients.



The surge added mass coefficients measured in the free oscillation range were consistent and ranged between 0.3 and 0.34 (Runs 17 - 24, Test 3862). The heave added mass coefficients ranged between 0.98 to 1.5 with an average of 1.14. Pitching added mass coefficients were found to exhibit higher variation 0.42 - 1.82 with an average of 1.29. The coupling coefficients obtained from the model tests were observed to be significantly higher than predicted by the literature or analytical model. The heave tests gave rise to pitching added mass coefficients of 0.75 - 1.94 with an average of 1.49. The pitching tests gave heave added mass coefficients of 0.87 - 1.27 with an average of 1.11.

For comparative purposes the 3-D finite element model added mass coefficients obtained from the comparison of dry and wet frequencies for both the skeletal and the complex structural/fluid idealisation of Reference (35) together with those obtained using the analytical approach of Appendix A are given in Table 16. The agreement between the open water finite element and the analytical formulae of Sec. 2.2 which particularly for the rigid body and flexural modes is quite good. It would be more appropriate to compare the skeletal results in the presence of an ice edge with the same model in open water conditions since this would directly provide the effect of the ice edge on the added mass.

It is clear from Table 16 that the skeletal model is somewhat stiffer than the more rigorous three dimensional model of Ref. (35). The added mass phenomenon is known to be frequency dependent and this, the presence of the ice edge plus the more detailed modelling of the fluid around the hull in the present project are considered to be the main reasons for the differences in the added mass coefficients between the skeletal and the previous fluid model.



TABLE 16  
Comparison of Added Mass Coefficients for Kigoriak

| Mode<br>No. | Mode   | Analytical<br>Model<br>(Secs. 2.2,2.3) | Full Scale<br>Data<br>(Sec. 2.3,3.3) | Model<br>Tests<br>(Sec. 2.4, 3.4) | Skeletal<br>Model*<br>(Sec. 2.5, 3.5) | 3D<br>(Ref.35) |
|-------------|--------|----------------------------------------|--------------------------------------|-----------------------------------|---------------------------------------|----------------|
| 1           | Surge  | 0.13                                   | 0.20                                 | 0.0 - 0.34                        | 0.082                                 | 0.09           |
| 2           | Heave  | 1.1                                    | 1.04<br>(Combined)                   | 0.98 - 1.94                       | 1.2                                   | 1.18           |
| 3           | Pitch  | 1.1                                    |                                      | 0.42 - 1.82                       | 1.1                                   | 1.9            |
| 4           | 2-node | 0.877                                  |                                      |                                   | 0.76                                  | 0.873          |
| 5           | 3-node | 0.795                                  |                                      |                                   | 0.58                                  | 0.762          |
| 6           | 4-node | 0.794                                  |                                      |                                   | 0.47                                  | 0.69           |

\* with an ice edge

### 3.7 Summary

The results presented in this section give a representation of the added mass coefficients in surge, heave and pitch and the first few flexible models for a ship in the ramming mode of icebreaking. The analytical model has shown that the added mass in surge is approximately 10 -20 per cent of the heave and pitch values. The heave and pitch added mass coefficients were found to be in the range 0.9 - 1.2 for both the Kigoriak and the M.V. Arctic with the Kigoriak giving slightly higher values. The added mass depends significantly on the bottom and ice edge proximities.

The added mass coefficients obtained from the full scale data analysis are very sensitive to instants and locations where small accelerations occur due to the approximations involved in the determination of the impact forces. The variation of added mass along the hull was found to be strongly time dependent. It was only manually possible to eliminate the erroneous results from the valid results. The procedure recommended for future analysis of full scale data consists of carrying out a Fourier Transform analysis on the signals to filter out the undesired frequencies and to plot both the distributed forces and acceleration and make sure before dividing the force by the acceleration that no instabilities will incur.

The model tests gave a valuable data base which showed that the added mass during impact is time dependent. The damping ratios were determined together with added mass coefficients from a set of free oscillation tests which showed that the added mass is influenced more significantly by the bottom and ice proximity than by ice thickness or ice strength. It is essential to perform further analysis of these data before more comprehensive conclusions could be reached.

The finite element method of analysis was shown to be suitable for predicting the added mass by modelling both the hull and the fluid and including the ice and bottom boundaries. Further computer runs are required for a more complete evaluation of the parametric relationships between added mass and ice proximity, wall proximity, bottom proximity and ship geometry. The calculated added mass coefficients (Sec. 2.3 and 3.3) for the rigid body modes and the flexible modes compare reasonably with the analytical predictions refer to Eqs. and Secs. of Sec. 2.2 and 3.2.

TABLE 6  
M.V. Arctic Model Tests

| TEST NO. 3845 |       |          |            |        |       |            |
|---------------|-------|----------|------------|--------|-------|------------|
| RUN NO.       | TYPE  | TIME (s) | RATE (F/S) | Bp (m) | V ( ) | Fmax (kgf) |
| 1             | TRIAL | 5.48     |            |        |       |            |
| 2             |       | 37.15    |            |        |       |            |
| 3             |       | 3.91     |            |        |       |            |
| 4             | SURGE | 35.29    | S          |        | 12    | 0          |
| 5             | PITCH | 12.89    | F          |        | 12    |            |
| 6             | HEAVE | 12.38    |            |        | 12    |            |
| 7             | HEAVE | 9.52     |            |        | 12    |            |
| 8             | PITCH | 13.78    | F          |        | 12    |            |
| 9             | SURGE | 8.77     | S          |        |       | 55         |
| 10            | PITCH | 11.74    | F          |        | 12    | 60.7       |
| 11            |       | 8.68     |            |        | 12    | 41.3       |
| 12            |       | 8.65     |            |        | 12    | 116.8      |
| 13            | SURGE | 39.05    |            |        | 30    | 148        |
| 14            | HEAVE | 10.55    |            |        | 30    |            |
| 15            | PITCH | 6.03     | F          |        | 30    |            |
| 16            | SURGE | 25.58    |            |        | 30    | 115        |
| 17            |       | 16.05    |            |        | 30    | 44.2       |
| 18            | PITCH | 22.14    | F          |        | 30    | 36.6       |
| 19            | SURGE | 12.92    | F          |        | 50    |            |
| 20            | HEAVE | 6.9      |            |        | 50    |            |
| 21            | PITCH | 7.18     |            |        | 50    |            |
| 22            |       | 7.65     |            |        | 50    |            |

TABLE 6 (Continued)

TEST NO. 3847

|    |       |         |     |    |       |
|----|-------|---------|-----|----|-------|
| 1  | PITCH | 5.9 F   | 100 | 0  |       |
| 2  |       | 10.9    |     | 5  |       |
| 3  |       | 8.45    |     | 10 |       |
| 4  |       | 5.1     |     | 0  |       |
| 5  | SURGE | 17.85   |     | 0  | 48.2  |
| 6  |       | 18.75   |     |    | 98.2  |
| 7  |       | 17.8    |     |    | 85.6  |
| 8  |       | 22.85   |     | 5  | 83.8  |
| 9  |       | 18.95   |     | 10 | 82.1  |
| 10 |       | 22.3    |     | 15 | 85.6  |
| 11 | HEAVE | 8.7     |     | 0  |       |
| 12 |       | 9.65    |     | 5  |       |
| 13 |       | 7.45    |     | 10 |       |
| 14 | PITCH | 7.1     |     | 0  | 55.44 |
| 15 |       | 7.55    |     | 5  | 56.2  |
| 16 |       | 11.7    |     | 10 | 64.7  |
| 17 |       | 12.55   |     | 10 | 64.6  |
| 18 |       | 10.85   |     | 10 | 52.7  |
| 19 |       | 6.8     | 50  | 0  | 51.1  |
| 20 | HEAVE | 6.2     |     |    |       |
| 21 |       | 17.1    |     |    |       |
| 22 | PITCH | 11.25   | 30  | 0  | 42.8  |
| 23 | HEAVE | 9.6     |     |    |       |
| 24 | PITCH | 9.9     | 12  |    | 50    |
| 25 |       | 6.25 S  |     |    | 37.6  |
| 26 | HEAVE | 7.3 F   |     |    |       |
| 27 |       | 10.2 S  |     |    |       |
| 28 | SURGE | 28.05   |     |    | 88.4  |
| 29 |       | 26.55 S |     |    | 26.7  |
| 30 |       | 19 S    |     |    | 37.7  |
| 31 |       | 25.85 F | 30  |    | 75.6  |
| 32 |       | 17.75   | 50  |    | 104   |

TABLE 6 (Continued)

| TEST NO. 3848 |       |         |      |       |        |       |           |  |
|---------------|-------|---------|------|-------|--------|-------|-----------|--|
| RUN NO.       | TYPE  | TIME(s) | RATE | h (m) | Bp (m) | x (m) | Fmax(kgf) |  |
| 1             | SURGE | 14.75   | F    | 15    | 30     | 20    | 105.8     |  |
| 2             | PITCH | 9.9     | F    |       |        |       | 54.8      |  |
| 3             | HEAVE | 7.8     |      | 15    |        |       |           |  |
| 4             | PITCH | 10.05   | F    | 6     |        |       | 56        |  |
| 5             | HEAVE | 11.1    | F    |       |        |       |           |  |
| 6             | SURGE | 16.1    | F    |       |        |       | 44        |  |
| 7             |       | 17.3    | F    |       |        |       | 63        |  |
| 8             | SURGE | 20.9    |      | 10    |        |       | 81.4      |  |
| 9             | HEAVE | 9.5     |      |       |        |       |           |  |
| 10            | SURGE | 24.1    | S    |       |        |       | 25.6 ?    |  |
| 11            | HEAVE | 10.45   | S    |       |        |       |           |  |
| 12            | SURGE | 28.6    | F    |       |        |       | 89.1      |  |
| 13            | HEAVE | 9.45    |      |       |        |       |           |  |
| 14            | SURGE | 21.2    |      |       |        |       | 81.9      |  |
| 15            | HEAVE | 10.6    |      | 10    |        |       |           |  |
| 16            | SURGE | 25.7    |      | 10    |        |       | 63.6      |  |
| 17            | HEAVE | 8.85    | F    | 10    |        |       |           |  |
| 18            | PITCH | 9.9     |      | 10    |        |       | 48.7      |  |
| 19            |       | 8.1     |      | 10    |        |       | 45.7      |  |
| 20            |       | 8.7     | F    |       |        |       | 50.8      |  |
| 21            |       | 9.55    | S    |       |        |       | 30.9      |  |
| 22            |       | 9.2     | F    | 10    |        |       | 49.8      |  |
| 23            |       | 8.45    |      |       |        |       | 52.2      |  |
| 24            | HEAVE | 9.7     |      |       |        |       |           |  |

TABLE 6 (Continued)

| TEST NO. 3855 |        |       |          |         |       |       |         |         |
|---------------|--------|-------|----------|---------|-------|-------|---------|---------|
| RUN NO.       | TYPE   | TIME  | SIG(MPa) | Hice(m) | Bp(m) | V(kn) | Fx(kgf) | Fz(kgf) |
| 1             | IMPACT | 20.84 | 3.5      |         |       | 30    | 5       | 18.8    |
| 2             |        | 18.18 |          |         |       |       | 10      | 35.7    |
| 3             |        | 9.3   |          |         |       |       | 15      | 133     |
| 4             |        | 10.68 |          |         |       | 12    | 5       | 18.2    |
| 5             |        | 7.86  |          |         |       |       | 10      | 57.2    |
| 6             |        | 10.04 |          |         |       |       | 15      | 150.9   |
| 7             |        | 13.34 |          |         |       | 50    | 5       | 25.9    |
| 8             |        | 9.26  |          |         |       |       | 10      | 50.5    |
| 9             |        | 8.96  |          |         |       |       | 15      | 148.7   |
| 10            |        | 11.92 |          |         | 5     | 50    | 5       | 23.2    |
| 11            |        | 7.92  |          |         |       |       | 10      | 52.4    |
| 12            |        | 7.6   |          |         |       |       | 15      | 152.75  |
| 13            |        | 11.18 |          |         |       | 30    | 5       | 34.5    |
| 14            |        | 8.72  |          |         |       |       | 10      | 54.3    |
| 15            |        | 7.38  |          |         |       |       | 15      | 152     |
| 16            |        | 11.12 |          |         |       | 12    | 5       | 30.3    |
| 17            |        | 7.7   |          |         |       |       | 10      | 57.5    |
| 18            |        | 8.02  |          |         |       |       | 15      | 152.3   |
| 19            |        | 10.66 |          |         | 15    | 12    | 5       | 30.4    |
| 20            |        | 9.28  |          |         |       |       | 10      | 51.1    |
| 21            |        | 8.36  |          |         |       |       | 15      | 134     |
| 22            |        | 11    |          |         |       | 30    | 5       | 35      |
| 23            |        | 7.96  |          |         |       |       | 10      | 46.3    |
| 24            |        | 7.56  |          |         |       |       | 15      | 142.5   |
| 25            |        | 10.12 |          |         |       | 50    | 5       | 25.9    |
| 26            |        | 8.68  |          |         |       |       | 10      | 45.1    |
| 27            |        | 7.38  |          |         |       |       | 15      | 130.5   |

TABLE 6 (Continued)

TEST NO. 3856

| RUN NO. | TIME(s) | SIG(MPa) | Hica(m) | Bp(m) | V (Kn) | Fx(kgf) | Fz(kgf) |
|---------|---------|----------|---------|-------|--------|---------|---------|
| 1       | 10.52   | 14       | 15      | 50    | 5      | 22.8    | 32.9    |
| 2       | 8.63    | 14       | 15      | 50    | 10     | 69.7    | 86.3    |
| 3       | 8.2     | 14       | 15      | 50    | 15     | 110     | 128.9   |
| 4       | 10      | 14       | 15      | 30    | 5      | 21.9    | 26.9    |
| 5       | 7.86    | 14       | 15      | 30    | 10     | 64.9    | 83.75   |
| 6       | 7.76    | 14       | 15      | 30    | 15     | 82      | 112.5   |
| 7       | 11.46   | 14       | 15      | 12    | 5      | 20.6    | 25.8    |
| 8       | 9.92    | 14       | 15      | 12    | 10     | 53.6    | 77.45   |
| 9       | 7.24    | 14       | 15      | 12    | 15     | 100     | 111     |
| 10      | 10.22   | 14       | 10      | 12    | 5      | 21.9    | 27.2    |
| 11      | 8.4     | 14       | 10      | 12    | 10     | 64.4    | 77.4    |
| 12      | 7.4     | 14       | 10      | 12    | 15     | 107.5   | 147.5   |
| 13      | 9.52    | 14       | 10      | 30    | 5      | 19.3    | 31      |
| 14      | 7.8     | 14       | 10      | 30    | 10     | 71.1    | 87.2    |
| 15      | 6.84    | 14       | 10      | 30    | 15     | 105     | 140     |
| 16      | 10.04   | 14       | 10      | 50    | 5      | 17.2    | 27.2    |
| 17      | 7.66    | 14       | 10      | 50    | 10     | 57.1    | 79.2    |
| 18      | 7.16    | 14       | 10      | 50    | 15     | 96.8    | 39      |
| 19      | 7.66    | 14       | 10      | 50    | 15     | 105     | 120     |
| 20      | 10.8    | 14       | 5       | 50    | 5      | 15.25   | 19      |
| 21      | 7.56    | 14       | 5       | 50    | 10     | 58.2    | 74.2    |
| 22      | 7.68    | 14       | 5       | 50    | 15     | 109     | 141.9   |
| 23      | 10.84   | 14       | 5       | 30    | 5      | 19.27   | 29.8    |
| 24      | 5.54    | 14       | 5       | 30    | 10     | 62.1    | 91.9    |
| 25      | 6.74    | 14       | 5       | 30    | 15     | 100     | 140     |
| 26      | 9.58    | 14       | 5       | 12    | 5      | 15.12   | 26.1    |
| 27      | 7.58    | 14       | 5       | 12    | 10     | 64.8    | 90.1    |
| 28      | 6.52    | 14       | 5       | 12    | 15     | 120     | 140     |



TABLE 6 (Continued)

TEST NO. 3857

|    |      |    |    |    |    |       |       |
|----|------|----|----|----|----|-------|-------|
| 1  | 8.62 | 35 | 6  | 30 | 5  | 21.9  | 18    |
| 2  | 8.08 |    |    |    | 10 | 52.95 | 61.1  |
| 3  | 6.44 |    |    |    | 15 | 145.2 | 119.9 |
| 4  | 6.76 |    | 6  | 30 | 15 | 142.2 | 123   |
| 5  | 9.34 |    | 10 |    | 5  | 16.4  | 13.9  |
| 6  | 7.98 |    |    |    | 10 | 52.5  | 52.4  |
| 7  | 6.46 |    | 10 | 30 | 15 | 146.6 | 122.1 |
| 8  | 9.62 |    | 15 |    | 5  | 17.05 | 12.3  |
| 9  | 8.36 |    |    |    | 10 | 60    | 49.3  |
| 10 | 6.84 |    | 15 | 30 | 15 | 153.9 | 134.9 |
| 11 | 9.38 |    |    | 30 | 5  | 11.7  | 9.9   |
| 12 | 8.44 |    |    |    | 10 | 48.4  | 52    |
| 13 | 6.12 |    |    |    | 15 | 200.3 | 176.3 |

TABLE 7  
M.V. Kigoriak Model Tests

| TEST NO. 3858 | RUN NO. | TYPE   | TIME  | SIG(MPa) | Hice(m) | Bp(m) | V(kn) | Fx(kgf) | Fz(kgf) |
|---------------|---------|--------|-------|----------|---------|-------|-------|---------|---------|
|               | 1       | IMPACT | 17.06 |          |         |       |       | 38      | 121.4   |
|               | 1       |        | 8.88  | 4.7      | 5       |       | 12    | 5       | 43.7    |
|               | 2       |        | 6.86  |          |         |       |       | 10      | 177.5   |
|               | 3       |        | 6.38  |          |         |       |       | 15      | 423.9   |
|               | 4       |        | 10.36 |          |         |       | 30    | 5       | 64.2    |
|               | 5       |        | 7.04  |          |         |       |       | 10      | 191.3   |
|               | 6       |        | 7.22  |          |         |       |       | 15      | 531.6   |
|               | 7       |        | 7.04  |          |         |       | 40    | 5       | 59.1    |
|               | 8       |        | 7.5   |          |         |       |       | 10      | 217.9   |
|               | 9       |        | 6.14  |          |         |       |       | 15      | 582.5   |
|               | 10      |        | 10.64 |          | 10      |       | 40    | 5       | 58.1    |
|               | 11      |        | 8.12  |          |         |       |       | 10      | 239.2   |
|               | 12      |        | 7.92  |          |         |       |       | 15      | 580.1   |
|               | 13      |        | 9.34  |          |         |       | 25    | 5       | 69.9    |
|               | 14      |        | 8.62  |          |         |       |       | 10      | 256.6   |
|               | 15      |        | 6.18  |          |         |       |       | 15      | 586.1   |
|               | 16      |        | 8.62  | 4.7      | 10      |       | 12    | 5       | 70.3    |
|               | 17      |        | 8.34  |          |         |       |       | 10      | 269.9   |
|               | 18      |        | 5.72  |          |         |       |       | 15      | 489.3   |
|               | 19      |        | 6.8   |          | 15      |       | 12    | 5       | 52      |
|               | 20      |        | 6.42  |          |         |       |       | 10      | 186     |
|               | 21      |        | 5.56  |          |         |       |       | 15      | 627.6   |
|               | 22      |        | 6.86  |          |         |       | 25    | 5       | 63.5    |
|               | 23      |        | 5.52  |          |         |       |       | 10      | 301     |
|               | 24      |        | 4.86  |          |         |       |       | 15      | 475.3   |
|               | 25      |        | 5.94  |          |         |       | 40    | 5       | 66.8    |
|               | 26      |        | 5.88  |          |         |       |       | 10      | 292.1   |
|               | 27      |        | 4.5   |          |         |       | 40    | 15      | 620.7   |
|               | 28      |        | 5.86  |          |         |       |       | 5       | 43.4    |
|               | 29      |        | 3.52  |          |         |       |       | 10      | 421.6   |
|               | 30      |        | 4.6   |          |         |       |       | 15      | 607     |

TABLE 7 (Continued)

| M.V. KIGORIAK IMPACT DEC. 08, 1986 50 Hz |         |          |         |       |        |         |         |
|------------------------------------------|---------|----------|---------|-------|--------|---------|---------|
| RUN NO.                                  | TIME(s) | SIG(MPa) | Hice(m) | Bp(m) | V (Kn) | Fx(kgf) | Fz(kgf) |
| 1                                        | 5.56    | 9.3      | 15      | 40    | 5      | 34      | 101.8   |
| 2                                        | 4.02    |          |         |       | 10     | 110     | 289.2   |
| 3                                        | 4.44    |          |         |       | 15     | 228     | 446.7   |
| 4                                        | 6.44    |          |         |       | 5      | 63.5    | 141.6   |
| 5                                        | 6       |          |         |       | 10     | 162.5   | 340.3   |
| 6                                        | 4.48    |          |         |       | 15     | 280     | 540.6   |
| 7                                        | 5.42    |          |         | 12    | 5      | 54.6    | 113.1   |
| 8                                        | 5.16    |          |         |       | 10     | 162.5   | 296.7   |
| 9                                        | 3.96    |          |         |       | 15     | 220     | 408     |
| 10                                       | 6.4     |          | 10      | 12    | 5      | 66.1    | 110.6   |
| 11                                       | 6.34    |          |         |       | 10     | 193.4   | 346.5   |
| 12                                       | 4.3     |          |         |       | 15     | 330     | 469.3   |
| 13                                       | 4.8     |          |         | 25    | 5      | 66.13   | 132.3   |
| 14                                       | 5.7     |          |         |       | 10     | 173.9   | 333.5   |
| 15                                       | 6.04    |          |         |       | 15     | 345     | 449.4   |
| 16                                       | 4.88    |          | 10      | 40    | 5      |         | 125.7   |
| 17                                       | 6.68    |          |         |       | 5      | 69.2    | 281.9   |
| 18                                       | 5       |          |         |       | 10     | 177.9   | 379.3   |
| 19                                       | 5.84    |          |         |       | 15     | 282.5   | 112.1   |
| 20                                       | 5.16    |          | 5       | 40    | 5      | 67.3    | 258     |
| 21                                       | 3.62    |          |         |       | 10     | 170     | 375     |
| 22                                       | 6.16    |          |         |       | 15     | 300     | 113     |
| 23                                       | 5.54    |          |         | 25    | 5      | 60.8    | 297.2   |
| 24                                       | 4.06    |          |         |       | 10     | 197.9   | 534     |
| 25                                       | 4.78    |          |         |       | 15     | 420     | 138.8   |
| 26                                       | 5.42    |          |         | 12    | 5      | 77.7    | 240     |
| 27                                       | 6.02    |          |         |       | 10     | 170     | 432     |
| 28                                       | 6.04    |          |         |       | 15     | 300     | 105.8   |
| 29                                       | 4.48    |          |         | 12    | 5      | 45.1    | 289.6   |
| 30                                       | 4.2     |          |         |       | 10     | 120     | 330     |
| 31                                       | 4.22    |          |         |       | 15     | 160     |         |

TABLE 7 (Continued)

[illegible]

TABLE 7 (Continued)

| TEST NO. 3861 |       |          |            |        |       |            |       |
|---------------|-------|----------|------------|--------|-------|------------|-------|
| -----         |       |          |            |        |       |            |       |
| RUN NO.       | TYPE  | TIME (s) | RATE (F/S) | Bp (m) | V (m) | Fmax (kgf) |       |
| 1             | SURGE | 25.25    | F          |        |       | 0          | 77.8  |
| 2             |       | 13.9     |            |        |       |            | 99    |
| 1             | SURGE | 11.65    | F          |        |       | 0          | 132.8 |
| 2             |       | 11.05    |            |        |       |            | 158.3 |
| 3             |       | 12.2     |            |        |       |            | 152.4 |
| 4             |       | 12.8     | S          |        |       |            | 77.1  |
| 5             |       | 11.3     | F          |        |       |            | 161   |
| 6             |       | 10.75    |            |        |       | 5          | 165.5 |
| 7             |       | 11.75    |            |        |       | 10         | 148.8 |
| 8             | PITCH | 7.1      | F          |        |       | 0          | 134   |
| 9             |       | 10.5     | S          |        |       |            | 76.7  |
| 10            |       | 9.85     | F          |        |       |            | 140.8 |
| 11            |       | 9.2      |            |        |       |            | 134.8 |
| 12            |       | 8.95     |            |        |       |            | 147.7 |
| 13            |       | 7.85     |            |        |       | 5          | 148   |
| 14            |       | 9.2      |            |        |       | 10         | 154.5 |
| 15            | HEAVE | 13.2     | F          |        |       | 0          |       |
| 16            |       | 6.8      |            |        |       |            |       |
| 17            |       | 7.75     |            |        |       |            |       |
| 18            |       | 8.4      | S          |        |       |            |       |
| 19            |       | 8.3      | F          |        |       |            |       |
| 20            |       | 8        |            |        |       |            |       |
| 21            |       | 9.6      |            |        |       | 5          |       |
| 22            |       | 9.1      |            |        |       | 10         |       |



## 4.0 CONCLUSIONS AND RECOMMENDATIONS

### 4.1 Introduction

The project presented in this report represents a pioneering effort directed to an area where no previous research work has been dedicated; namely the evaluation of the hydrodynamic added mass for ships ramming ice. For approximately twenty years, researchers have always assumed that open water added mass coefficient determined from ship vibration laboratories are valid for ice ramming. Recent full scale impact test data have shown that the added mass might be highly variable both with time during the impact scenario and with location along the hull girder. The prediction of ice forces exerted on the hull of icebreaking ships is the governing design parameter that controls both the local and global strengthening requirements. The added mass by definition is added to the mass of the ship to represent the effect of the surrounding water, and will therefore have a direct bearing on the magnitude of the developed ramming forces. This demonstrates the justification that lead to the present study with the objective of developing an analytical model for evaluating added mass for ships ramming ice.

The following two subsections present the salient conclusions drawn from work carried out and recommendations made for further work.

### 4.2 Conclusions

The methodology and results described in Sections 2 and 3 have included four main areas; namely analytical modelling, full scale data analysis, model testing, and finite element modelling. The conclusions drawn from this work may be divided into general and specific conclusions and are summarized in the following.

#### 4.2.1 General Conclusions

The work carried out herein represents a forward step in this challenging area of ship ice interaction.



The following general conclusions were drawn from the work:

1. The present state-of-the-art is inadequate to provide a full description of the added mass for all phases of the impact process; namely the approach, impact and beaching phases. The presently available models (Sec. 2.2) give reasonable results for the open water behaviour.
2. Analytical models (Sec. 2.2) based on ideal flow theory have been presented and are shown to give results comparable with the state-of-the-art. These models are not yet ready for direct application and require further development.
3. The analytical investigations (Sec. 2.2) have yielded useful conclusions regarding the identification of the most significant parameters influencing the added mass.
4. The surge added mass coefficient was found to be in order of 10 - 20 percent of the heave and pitching added mass coefficients which are of the same order of magnitude.
5. The surge-heave and surge-pitch coupling coefficients are negligible in comparison with the heave-pitch coupling coefficient which was found to be significant.
6. The added mass coefficients are strongly dependent on the waterline coefficient and the beam/draft and beam/length ratios.
7. Ships having bluff bow and hull geometry exhibit higher added mass coefficients than ships with more flared geometries.
8. The added mass varies considerably during the time (See Sec. 3.4) history of the ram. This time dependence requires further investigation in order to be fully understood.
9. The added mass can be measured in full scale testing by employing an array of triaxial gyro stabilized accelerometers along the length of the ship in addition to hull strain gauges to measure the bow impact force.
10. The vertical added mass is more influenced by the bottom proximity (water depth less the ship draft) than by the distance to the ice floe.
11. The ice thickness and ice strength do not seem to have any significant influence on the added mass.

12. It was possible to manually obtain added mass coefficients from the full scale data analysis. This was due to the reliance on the acceleration measurements at only two points along the hull, at the bow and the stern, to determine the acceleration distribution along the vessel length. The errors in the estimated forces combined with the near zero accelerations give rise to unrealistically high added mass coefficients.
13. The model testing program was successful in obtaining an extensive data base which is yet to be fully analysed. Approximately 30% of this data was partially analysed for the purpose of the present report. It must be emphasized that full analysis of this data is essential in order to further our understanding of this problem.
14. Similar to the full scale data analysis, problems were encountered when analysing the model test data. The heave added mass values were considerably influenced by the small accelerations and also showed unstable behaviour.
15. The model test data gave more reliable results for the pitch added mass values of the ramming tests. Reasonable added mass coefficients were calculated for all rigid body motions in the free oscillation range of the tests.
16. The finite element method of analysis was successfully employed to calculate the added mass distribution along the hull. This general method is believed to be the only theoretical way available which can efficiently include the effects of hull shape, ice and bottom boundaries of the geometry.
17. The Det norske Veritas software package SESAM was employed in the three dimensional modelling of the Kigoriak, the surrounding fluid and the ice and bottom boundaries.
18. More computer runs are required before conclusive results can be obtained and applied to the development of a reliable analytical model.

#### 4.2.2 Specific Conclusions

The following specific conclusions were drawn from the four distinct areas of analytical modelling, full scale data analysis, model testing, and finite element modelling.

The conclusions drawn are summarised as follows:

1. The mass of an accelerating ship is increased by the presence of water surrounding the ship. This increase is called "added mass" and manifests itself in a change of hydrodynamic pressure acting on the hull.
2. The change of pressure due to the added mass effect is proportional to acceleration of the ship, water density, and the linear dimensions of the ship.
3. The added mass coefficient, in general, can be expressed as
$$\lambda = c \times A^{3/2} / \nabla$$
where  $c$  is a constant,  $A$  is the area projected normal to the direction of motion; and  $\nabla$  is the volume of the body.
4. The added mass coefficients of the ship can be calculated as a first approximation, using a flat plate analogy. This analogy gives lower bound values of added mass. This flat plate analogy can be used for studying the added mass of ship's components such as bow, stern, appendages, etc.
5. The state of the art expressions for the added mass of icebreaking ships recommended for the comparison with the full scale and model scale test results are given by Eq. 12 - 14.
6. The developed added mass coefficients and their parameter dependence are given by Eqs. 15 to 19.
7. For the M.V. Canmar Kigoriak and the M.V. Arctic, the calculated values of these added mass coefficients are (Sec. 3.2):

| <u>Rigid Body Motion</u> | <u>Kigoriak</u>   |                 | <u>Arctic</u>     |                 |
|--------------------------|-------------------|-----------------|-------------------|-----------------|
|                          | <u>Literature</u> | <u>Proposed</u> | <u>Literature</u> | <u>Proposed</u> |
| Surge                    | 0.075             | 0.13            | 0.075             | 0.05            |
| Heave                    | 1.07              | 1.1             | 0.81              | 0.91            |
| Pitch                    | 0.91              | 1.04            | 0.77              | 0.80            |

8. Added mass distribution in flexural vibrations can be expressed as given by Eq. 21 as function of the submerged cross sectional area, the width and draft of the section, the water density, and the length to beam ratio. For the M.V. Canmar Kigoriak, Equation 21 gives an added mass density in the first flexural mode of 71.2 tonnes/m.
9. The investigation of the cross-coupling of added masses has shown that for an icebreaking ship, the significant cross coupling occurs between heave and pitch.
10. The added mass of a ramming ship can be expressed as (Eq. 18)

$$\lambda = \lambda^0 \times K_{bp} \times K_{wp} \times K_{fs} \times K_{fm} \times K_{sp}$$

where the coefficients K expresses the corrections due to the effects of bottom proximity, wall proximity, free surface, frequency of motion, and space.

Analytical computations of the correction factors suggest that these have magnitudes of the order of 20% to 40%.

11. The state of the art understanding of the added mass phenomenon suggests that the added mass coefficients in free motions, the impact phase, and the beaching phase should be constant except for the differences caused by changes in the relative geometry of the ship/ice floe during the ramming.
12. These results of the model test date analysis show very large fluctuations of added mass coefficients within each test and from one test to another. The comparison of the model test results with the analytical computations show a reasonably good agreement for free motions. However, the large fluctuations of the test results during the impulse and ramming tests suggests that there is a source of instability in either the signals or the computational method.

13. The model tests did yield reasonable surge or heave added mass coefficients for the impact phase. The pitch added mass coefficient of 0.5 to 1.325 includes most of the data points. It was not possible to calculate added mass coefficients during the beaching phase due to the small accelerations measured during beaching and the presence of a vibratory component in the acceleration signal. The impulse tests gave more reasonable values for the added mass coefficients. The maximum surge added mass coefficient measured was 0.1. The heave added mass coefficients of 1.66 was measured due to an impulse applied in the bow. The max pitch added mass coefficients measured was 0.95. The values were estimated at the time of the peak force during the impulse. The declinations tests (Forced motions or Impulse tests) however gave much more reasonable values for the added mass coefficients.
14. The surge added mass coefficients measured in the free oscillations range were consistent and ranged between 0.3 and 0.34 (Runs 17 - 24, Test 3862). The heave added mass coefficients ranged between 0.98 to 1.5 with an average of 1.14. Pitching added mass coefficients were found to exhibit higher variation 0.42 - 1.82 with an average of 1.29. The coupling coefficients obtained from the model tests were observed to be significantly higher than predicted by the literature or analytical model. The heave tests gave rise to pitching added mass coefficients of 0.75 - 1.94 with an average of 1.49. The pitching tests gave heave added mass coefficients of 0.87 - 1.27 with an average of 1.11.
15. For comparative purposes the 3-D finite element model added mass coefficients obtained from the comparison of dry and wet frequencies for both the skeletal and the complex structural/fluid idealisation of Reference (35) together with those obtained using the analytical approach of Appendix A are given in Table 16. The agreement between the open water finite element and the analytic approaches particularly for the rigid body and flexural modes is quite good. It would be more appropriate to compare the skeletal results in the presence of an ice edge with the same model in open water conditions since this would directly provide the effect of the ice edge on the added mass.

16. It is clear from Table 16 that the skeletal model is somewhat stiffer than the more rigorous three dimensional model of Ref. (35). The added mass phenomenon is known to be frequency dependent and this, the presence of the ice edge plus the more detailed modelling of the fluid around the hull in the present project are considered to be the main reasons for the differences in the added mass coefficients between the skeletal and the previous fluid model.



### 4.3 Recommendations

The work carried out in this project has resulted in several developments which are new in this field. However, a full realization of the benefits of the results of this project requires further work. The following recommendations are therefore made to specify the areas where further work would be most productive:

1. A full analysis of the model test data collected should be made. This analysis would give a more accurate evaluation of the added mass coefficients and their time dependence and dependence on the parameters identified; namely bottom proximity, distance to floe, velocity, floe thickness and floe strength. The effect of friction should also be studied. Also, a Fast Fourier Transform analysis of the signals is recommended to identify the significant frequencies involved. A statistically based error analysis should also be carried out in order to predict the level of accuracy of the results. Statistical analysis of the data is also recommended.
2. Analysis of the M.V. Arctic full scale data, similar to that conducted for the Kigoriak in the present study is recommended in order to fully compare the results for the two ships.
3. Further finite element runs of the Kigoriak model for various ice edge proximities thicknesses are required in order to more reliably determine the added mass dependence on the parameters identified in this study.
4. Finite element modelling of the M.V. Arctic is recommended in order to complete the comparison between the two vessels since only the Kigoriak was modelled in this study.
5. In tandem with 3 above it would be extremely valuable and cost effective to model simple form vessels varying only the bow shape and the hull dimensions and ice edge conditions.
6. Refinement of the analytical model is required. Such refinement should be based on a more in depth analysis of the physical process of added mass.
7. Further model tests using different hull forms would yield significant data that is needed for a more complete understanding of the dependence of the



added mass on the bow shape and the size of the vessel. It should also be attempted to model the ice behaviour during impact in these tests.

8. Damping has only been evaluated here for the free oscillation model tests. It would be of interest to determine the damping forces acting on the ship during the impact and beaching phases in order to confirm the expectation that damping forces are not significant during the ramming phase.
9. Investigation of the influence of scaling laws on the prediction of added mass for large vessels.
10. Development of a software package capable of interactively predicting the added mass and ship ice interaction forces for any ship ramming ice.
11. Full scale testing is recommended in order to evaluate the added mass through the accurate measurement of the interaction forces and the accelerations throughout the interaction and along the length of the ship.

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